On Inter-cell Interference Factor In the Uplinks of Multicell Planar Networks

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***Abstract*—It is of great importance for service providers to evaluate the impact of inter-cell co-channel interference on the achievable data rate in cellular networks. In this paper, the cumulative distribution function (CDF) of the inter-cell interference factor is derived. On this basis, closed-form expression of the maximum achievable rate subject to the interference from finite planar cells is analyzed over Nakagami-m fading uplink channels. Numerical results show that besides Nakagami-m fading severity parameters and path loss coefficient, the maximum achievable rate is also affected by the number of interference cells. According to the numerical results, with an increase in the number of interference cells, the maximum achievable rate is degraded.**

***Index Terms*—Maximum achievable rate, Nakagami-m fading, path loss.**

I. Introduction

Interference and fading are two key challenges for successful communication in wireless systems. In cellular communication systems, interference arises when different base stations share the same carrier frequency due to frequency reuse. Inter-cell interference reduces data rates throughout the cells and even causes outages at the cell edges. Therefore, it is of great importance and interests for researchers and network operators to evaluate the impact of inter-cell co-channel interference on the achievable data rate in cellular networks.

Various models have been proposed to investigate the uplink capacity with co-channel interference [1]–[7]. A linear array cells model, i.e., the Wyner model, was proposed in [1] to analyze the capacity of cellular networks. The concept of inter-cell signal interference factor was defined to evaluate the impact of interference on the uplink capacity of cellular networks. In [2], the inter-cell signal interference factor is demonstrated to be location-dependent and the uplink maximum achievable rate per cell impacted by the user locations was analyzed. For the two-dimension planar model, a mathematical framework was introduced in [3] to characterize the network interference. Assuming that the interferers are distributed according to a spatial Poisson process, the statistical distribution of the aggregate interference was determined and the interference analysis was investigated in some applications. In [4], the authors assumed that the mobile user's location follows a homogeneous two-dimensional spatial Poisson Point Process (PPP) and each mobile user communicates to its closest base station. Then based on the Laplace transform of the interference, a novel technique using point processes to get easy-to-evaluate integral expressions was proposed. In [5], the applicability and accuracy of the linear Wyner model in uplink and downlink channels was evaluated. The uplink capacity is limited to scenarios where the interference can be spatially averaged, for example in a CDMA network under high load. The hexagonal grid model, where the base stations are usually deployed by deterministic grid, could not lead to a tractable framework in general. Thus the uplink capacity was derived using several simplifying approximations followed by exhaustive Monte Carlo simulations [6]. The co-channel interference from the adjacent cell was evaluated and the capacity gain by the cooperative cell network was analyzed in [7].

In the aforementioned work on characterizing the capacity of cellular networks, the Wyner model assumed that the received signal at each cell site is interfered by the active users from the adjacent cells only. As indicated in [4], the Laplace transform of the interference is only applicable in the traditional Rayleigh fading channels where user locations obey the spatial PPP. Motivated by these observations, in this paper we study the interference from finite planar cellular networks with Nakagami-m fading uplink channels. Taking into account the inter-cell signal interference factors, we investigate the maximum achievable rate in the uplink channels in this paper.

The rest of this paper is organized as follows. Section II describes the system model. In Section III, considering practical interference from finite cells, a closed-form expression of the maximum achievable rate per cell is derived for planar cellular networks with Nakagami-m fading uplink channels. In Section IV, numerical simulations show the impact of the number of interference cells, the path loss coefficient and Nakagami-m fading severity parameter on the maximum achievable rate. Finally, Section V concludes this paper and future work is prospected.

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II. System Model

Consider a planar model of M cells with one base station (BS) and K users per cell. The radius of each cell is assumed to be R. Users are assumed to follow an independent and identical uniform distribution throughout a cell. Further, it is assumed that the received signal at the base station in each cell is interfered by the active users from all other cells over the plane. This model is known as the planar model, as illustrated in Fig.1.

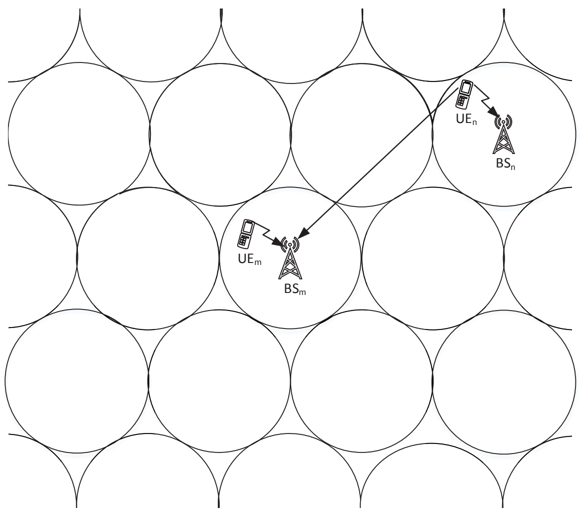

Fig. 1. A general planar model.

Taking into account the impact of the interference and noise in the planar model shown in Fig.1, the maximum achievable rate with single cell processing scheme in the cell is given by [1]

$$R_{SCP\_m} = E\left\{\log\left(1 + \frac{P_m}{N_m + I_m}\right)\right\} \quad (1)$$

where P_m is the instantaneous received signal power, I_m is the instantaneous interference power from other cells, N_m is the instantaneous noise power, and $E\{\cdot\}$ is an expectation operator.

Based on the simple one-slope path loss model, the received power at BS_m can be expressed as [9]

$$P_m = \frac{\Re \bar{P}_{tr}}{K} \sum_{k=1}^{K} (d_0/d_{m,k})^{\lambda} |a_{m,k}|^2 \quad (2)$$

where $d_0 = 1$ meter denotes the protected distance, λ denotes an attenuation coefficient, typically ranging from 2 to 5 , $\Re = -31.54dB$ denotes the propagation coefficient. P_m and $\bar{P}_{tr}$ are defined as the receiver power at BS_m and the average transmission power of each user in the local cell.

To avoid the near-far effect in cellular networks, every user is assumed to achieve perfect channel inversion with respect to user location in the uplink, i.e., the received signal power of each user at its home BS with fading is represented by a common value P. That is

$$P = \Re P_{tr\_k}(d_0/d_k)^{\lambda} \quad (3)$$

where P_{tr_k} denotes the transmission power of the k^{th} user in a cell, and d_k denotes the distance between the k^{th} transmitter and the receiver.

Then the instantaneous interference power from users in the n^{th} cell to the BS in the m^{th} cell can be written as

$$\begin{aligned} I_n &= \frac{\Re \bar{P}_{tr\_n}}{K} \sum_{t=1}^{K} \left(d_0/d_{n\_m,t}\right)^{\lambda} |b_{n\_m,t}|^2 \\ &= \frac{P}{K} \sum_{t=1}^{K} \left(d_{n,t}/d_{n\_m,t}\right)^{\lambda} |b_{n\_m,t}|^2 \end{aligned} \quad (4)$$

The noise is assumed as a zero mean, i.i.d. Gaussian process. The average power of noise in uplink channel is normalized to a unit power for simplicity. Substituting equation (3) and (4) into equation (1) and assuming perfect channel inversion, the maximum achievable rate R_{SCP_m} can be rewritten as

$$R_{SCP\_m} = E\left\{\log\left(1 + \frac{\frac{P}{K}\sum_{k=1}^{K}|a_{m,k}|^2}{1 + \frac{P}{K}\sum_{n=2}^{\infty}\sum_{t=1}^{K}\left(d_{n,t}/d_{n\_m,t}\right)^{\lambda}|b_{n\_m,t}|^2}\right)\right\} \quad (5)$$

From equation (5), it can be seen that the maximum achievable rate is affected by the channel fading parameters, the path loss from intended users to its home BS and to the BS of the m^{th} cell.

III. The interference factor in planar cellular networks

In this section, we analyze the interference factor that characterizes the path loss of interfering signals from users in all other cells to the m^{th} cell.

A. The cumulative distribution function of interference factor

The interference factor from the users in the n^{th} cell can be depicted in Fig. 2. The distance between the BSs in the m^{th} cell and the n^{th} cell is iR, i is the distance factor. the locations of users in each cell are assumed to follow a uniform distribution, the distance between $UE_{n,t}$ and BS_n is r , and the angle subtended by $UE_{n,t}$, BS_n and BS_m is denoted as θ.

From Fig.2, the interference factor of the t^{th} user $UE_{n,t}$ in the n^{th} cell can be written as $\alpha = d_{n,t}/d_{n\_m,t} = \frac{r}{\sqrt{(r\cos\theta - iR)^2 + r^2\sin^2\theta}}$.

Since users are assumed to follow a uniform distribution in a cell, the cumulative distribution function (CDF) of α can be written as

$$F_{\alpha}(\alpha) = \iint\limits_{\frac{r}{\sqrt{(r\cos\theta - iR)^2 + r^2\sin^2\theta}} \leqslant \alpha} \frac{r}{\pi R^2} dr d\theta \quad (6)$$

where the condition $\frac{r}{\sqrt{(r\cos\theta - iR)^2 + r^2\sin^2\theta}} \le \alpha$ can be rewritten as

$$\left(\alpha^2 - 1\right) r^2 - 2i\alpha^2 Rr\cos\theta + i^2\alpha^2 R^2 \ge 0 \tag{7}$$

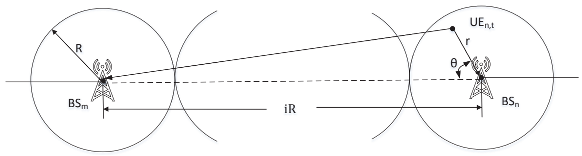

Fig. 2. The model of interference factor affected by distance factor.

Considering that r is the distance from a user to the intended BS in a cell, r is limited by

$$0 < r \le R \tag{8}$$

From equation (7) and (8), the CDF of α can be calculated in three cases.

1) $\alpha \le \frac{1}{i+1}$. In this case, the CDF of α can be derived as

$$\begin{aligned} F_\alpha(\alpha) &= \int_0^\pi \int_0^{\frac{\alpha^2 iR\cos\theta - \alpha iR\sqrt{1-\alpha^2\sin^2\theta}}{(\alpha^2-1)}} \frac{2r}{\pi R^2} dr d\theta \\ &= \frac{4i^2\alpha^2}{(\alpha^2-1)^2} \end{aligned} \tag{9a}$$

2) $\alpha > \frac{1}{i-1}$. In this case, the CDF of α can be derived as

$$F_\alpha(\alpha) = \int_0^\pi \int_0^R \frac{2r}{\pi R^2} dr d\theta = 1 \tag{9b}$$

3) $\frac{1}{i+1} < \alpha \le \frac{1}{i-1}$. In this case, the CDF of α can be derived as

$$\begin{aligned} F_\alpha(\alpha) = &\int_0^{g(\alpha)} \int_0^{\frac{\alpha^2 iR\cos\theta - \alpha iR\sqrt{1-\alpha^2\sin^2\theta}}{(\alpha^2-1)}} \frac{2r}{\pi R^2} dr d\theta \\ &+ \int_{g(\alpha)}^\pi \int_0^R \frac{2r}{\pi R^2} dr d\theta \\ =&1 + \frac{\alpha^2 i\sin(g(\alpha))}{\pi(\alpha^2-1)} - \frac{\alpha^2 i^2\arcsin[\alpha\sin(g(\alpha))]}{\pi(\alpha^2-1)^2} \\ &- \frac{g(\alpha)}{\pi} + \frac{\alpha^2 i^2 g(\alpha)}{\pi(\alpha^2-1)^2} \end{aligned} \tag{9c}$$

where $g(\alpha) = \arccos\frac{(\alpha^2-1)+\alpha^2 i^2}{2\alpha^2 i}$.

The CDF of the interference factor with respect to the distance coefficient i is illustrated in Fig. 3. From Fig. 3, it can be seen that the value of CDF curve with $i = 2$ approaches 1 at $\alpha = 1$. The reason is that in the case of $i = 2$, the m^{th} cell and the n^{th} cell is adjacent, thus only when the interfering user is located at the point of intersection of these two cells, the interference factor is 1. However, when the distance coefficient i is greater than 2 which means the n^{th} cell is not adjacent to the m^{th} cell, the CDF of α approaches 1 when α approaches $\frac{1}{i-1}$ where $\frac{1}{i-1} \le 1$. The reason is that in the case that the user is located in the n^{th} cell that is far away from the m^{th} cell, the interference factor is less than $\frac{1}{i-1}$.

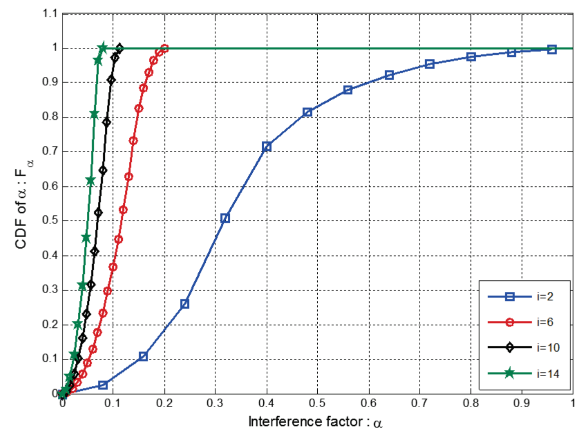

Fig. 3. The CDF of interference factor with different distance factors i.

B. Maximum Achievable Rate with Nakagami-m Fading Channels

To facilitate the derivation of the maximum achievable rate, we let $S = \sum_{k=1}^{K} |a_{m,k}|^2$, and $T_n = \sum_{t=1}^{K} |b_{n\_m,t}|^2$. The maximum achievable rate in (5) can thus be written as

$$\begin{aligned} R_{SCP\_m} =&E\left\{\log\left(K + P\left(S + \sum_{n=2}^{\infty} T_n \cdot \sum_{t=1}^{K}\left(d_{n,t}/d_{n\_m,t}\right)^\lambda\right)\right)\right. \\ &\left. - \log\left(K + P\sum_{n=2}^{\infty} T_n \cdot \sum_{t=1}^{K}\left(d_{n,t}/d_{n\_m,t}\right)^\lambda\right)\right\} \end{aligned} \tag{10}$$

Assuming that the uplink channel fading is independent of the interference factor, then (10) can be simplified using the average interference

$$\begin{aligned} R_{SCP\_m} =&E\left\{\log\left(K + P\left(S + \sum_{n=2}^{\infty} T_n \cdot [\bar{\alpha}_n(i)]\right)\right)\right. \\ &\left. - \log\left(K + P\sum_{n=2}^{\infty} T_n \cdot [\bar{\alpha}_n(i)]\right)\right\} \end{aligned} \tag{11}$$

where $\bar{\alpha}_n(i) = \sum_{t=1}^{K}\left(d_{n,t}/d_{n\_m,t}\right)^\lambda$ is the average interference factor from K users of the n^{th} cell that can be obtained from equation (9).

The Nakagami-m fading experienced by the desired signal in the m^{th} cell and the interference signals from other cells, which are represented by $|a_{m,k}|^2$ and $|b_{n\_m,t}|^2$, can be expressed by Gamma distributions [8], those are $|a_{m,k}|^2 \sim \Gamma(m_1, \Omega/m_1)$ and $|b_{n\_m,t}|^2 \sim \Gamma(m_n, \Omega/m_n)$ respectively, where m denotes the fading severity parameter and Ω denotes the received signal power at the corresponding BS.

The average interference factor caused by the users in an interference cell is constant. The Nakagami-m fading effect on the desired signal in the m^{th} cell is assumed to be independent of the effect on the interference signals from other cells. According to scaling property of gamma distribution from [9], the distributions of random variables S and T_n can be respectively obtained as

$$S = \sum_{k=1}^{K} |a_{m,k}|^2 \sim \Gamma\left(Km_1, \Omega/m_1\right) \tag{12a}$$

$$T_n \cdot \left[\bar{\alpha}_n\left(i\right)\right] = \left[\bar{\alpha}_n\left(i\right)\right] \sum_{t=1}^{K} |b_{n\_m,t}|^2 \sim \Gamma\left(Km_n, \left[\bar{\alpha}_n\left(i\right)\right]\Omega/m_n\right) \tag{12b}$$

There are different closed-form solutions for the sum of independent Gamma random variables [10]. For a practical interference model, since the interference factor from the users in the cell which is far enough can be ignored, the sum over infinite interference factors in the uplink channels can be truncated. Thus $\sum_{n=2}^{N} T_n \cdot \left[\bar{\alpha}_n\left(i\right)\right]$, which is the sum of $N-1$ independent Gamma variables, is used to represent the truncated interference.

In [11], a moment matching approximation of a finite number of independently distributed random variables was proposed. This technique involves approximating the sum of Gamma variables with another Gamma random variable with the same first and second order moments, and is rather standard in the statistics community as a way to approximate more complex distributions.

By using the moment matching technique, the sum of $N-1$ independent Gamma random variables can be approximated by a Gamma distribution $\Gamma\left(a_y, b_y\right)$ with the first and second order moments

$$a_y = \frac{K\left(\sum_{n=2}^{N}\left[\bar{\alpha}_n\left(i\right)\right]\right)^2}{\sum_{n=2}^{N}\frac{\left[\bar{\alpha}_n(i)\right]^2}{m_n}}, b_y = \frac{\Omega\sum_{n=2}^{N}\frac{\left[\bar{\alpha}_n(i)\right]^2}{m_n}}{\sum_{n=2}^{N}\left[\bar{\alpha}_n\left(i\right)\right]} \tag{13}$$

Then $S + \sum_{n=2}^{\infty} T_n \cdot \left[\bar{\alpha}_n\left(i\right)\right]$ can be approximated by a Gamma distribution $\Gamma\left(a'_y, b'_y\right)$ with

$$a'_y = \frac{K\left(1 + \sum_{n=2}^{N}\left[\bar{\alpha}_n\left(i\right)\right]\right)^2}{\left(\frac{1}{m_1} + \sum_{n=2}^{N}\frac{\left[\bar{\alpha}_n(i)\right]^2}{m_n}\right)}, b'_y = \frac{\Omega\left(\frac{1}{m_1} + \sum_{n=2}^{N}\frac{\left[\bar{\alpha}_n(i)\right]^2}{m_n}\right)}{\left(1 + \sum_{n=2}^{N}\left[\bar{\alpha}_n\left(i\right)\right]\right)} \tag{14}$$

Substituting (13) and (12a) into (11), a new maximum achievable rate with Nakagami-m fading uplink channels can thus be derived.

To simplify the expression, we resort to some special functions, e.g., the Meijer's G-functions [12]. The functions of $\log_2\left(\cdot\right)$ and $\Gamma\left(\cdot,\cdot\right)$ can be expressed as a special form of Meijer's G-function. Therefore, (11) can be expressed in a closed form using Meijer's G-functions

$$\begin{aligned} R_{SCP\_m} =& \frac{1}{\Gamma\left(a'_y\right)} \cdot \log e \cdot G_{3,2}^{1,3}\left({}^{1-a'_y,1,1}_{1,0}\middle| \frac{P}{K}b'_y\right) \\ &- \frac{1}{\Gamma(a_y)} \cdot \log e \cdot G_{3,2}^{1,3}\left({}^{1-a_y,1,1}_{1,0}\middle| \frac{P}{K}b_y\right) \end{aligned} \tag{15}$$

where a_y, b_y, ${a_y}'$ and ${b_y}'$ can be obtained from (13) and (14).

IV. Numerical Analysis

Based on the maximum achievable rate of Nakagami-m fading uplink channels obtained in the last section, some numerical performance evaluations are performed. The following parameters are used in the numerical evaluation: the received signal power Ω at a BS is normalized as 1, the average user transmission power is set as $\bar{P}_{tr} = 10$dB.

1) The effect of the path loss coefficient

Let $K = 1$ for the number of users in a cell, $m_1 = 1$ for the Nakagami-m fading severity parameter of the desired signal and $m_2, \cdots, m_N = 2$ for the Nakagami-m fading severity parameter of the interference. Since only the adjacent cells of the m^{th} cell are considered, we have $N = 7$. Then the maximum achievable rate with different path loss coefficients is shown in Fig. 4. It can be observed that the maximum achievable rate per cell decreases with an increase in the inter-cell signal interference factor. On the other hand, when the interference factor is fixed between 0 and 1, the maximum achievable rate per cell increases with an increase in the path loss coefficient λ from the interference cells.

2) The effect of the signal fading severity parameter

Similarly, we let $K = 1$, $N = 7$, and let $\lambda = 4$ for the path loss coefficient. For the desired signal the fading severity parameter is set as $m_1 = 1$, then the maximum achievable rate under different fading severity parameter $m_2, \cdots, m_N$ for the interference signal is shown in Fig. 5. It can be observed that the maximum achievable rate per cell decreases slightly with an increase in the fading severity parameter when the interference factor is fixed. This is reasonable as the change of the fading severity parameter severely affects the maximum achievable rate.

3) The effect of the number of interference cells

Similarly, we let $K = 1$, $m_1 = 1$, $m_2, \cdots, m_N = 2$, and $\lambda = 4$, then the maximum achievable rate with respect to the number of interference cells is shown in Fig. 5. It can be observed that the maximum achievable rate per cell decreases with an increase in the number of interference cells. This is reasonable as with an increase in the number of interference cells, the aggregate interference power is increased, thus degrading the maximum achievable rate. However, with more interference cells considered, since the interference power decreases with the distance, the degradation in the maximum achievable rate becomes slower. To be specific, the degradation

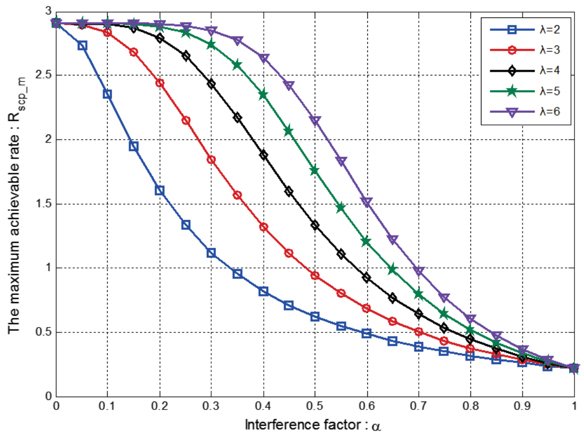

Fig. 4. The maximum achievable rate per cell with respect to the interference factor under different path loss coefficients.

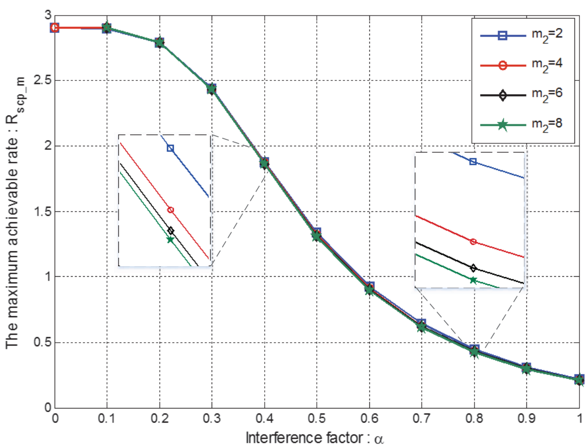

Fig. 5. The maximum achievable rate per cell with respect to the interference factor under different fading severity parameters.

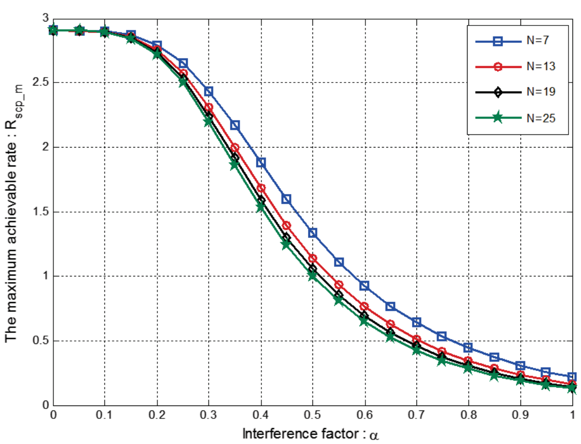

Fig. 6. The maximum achievable rate per cell with respect to the number of interference cells.

in the maximum achievable rate from $N = 13$ to $N = 19$ is smaller than that from $N = 7$ to $N = 13$.

V. Conclusions

In this paper, we derived the CDF of the inter-cell interference factor and obtained a closed-form expression of the maximum achievable rate in cellular networks with Nakagami-m fading uplink channels. Numerical results showed that the maximum achievable rate is affected by Nakagami-m fading severity parameters and path loss coefficient of the interference cells. Besides, with an increase in the number of interference cells, the maximum achievable rate per cell is degraded. Our analysis and results provide some insights in analyzing the planar wireless cellular networks and are benefit for upcoming 4G/5G systems. In the future work, we will further explore the gap between the planar regular cell model and the Poisson Voronoi Tessellation cell model in terms of the maximum achievable rate.

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