On the Performance of LTE/Wi-Fi Dual-Mode Uplink Transmission: Connection Probability versus Energy Efficiency

Jing Zhang, Jing Han, Lin Xiang, Derrick Wing Kwan Ng, Min Chen, and Minho Jo

Abstract—This paper evaluates the performance of uplink transmission in an LTE/Wi-Fi coexisting heterogeneous network (HetNet), where user equipments (UEs) capable of jointly performing LTE/Wi-Fi dual access and dual power control have a higher preference in connecting the Wi-Fi access points (WAPs) than the LTE base stations (LBSs). Thereby, each UE competes for Wi-Fi connection using carrier-sense multiple access with collision avoidance (CSMA/CA) and, only if unsuccessful, employs cellular communication. During uplink transmission, the UEs employ truncated channel inversion power control with receiving thresholds adjusted according to the occupied communication modes. Based on the stochastic geometry theory, we derive the analytical expressions for the successful connection probability and the average energy efficiency (EE) of Wi-Fi communications. We show that there exists a maximal successful connection probability for Wi-Fi communications, where the optimal receiving signal and energy thresholds of WAPs depend critically on the density of WAPs. Moreover, the average EE of UEs successfully accessing WAPs saturates in the large receiving signal threshold regime, independent of the density of WAPs. These findings significantly differ from those in cellular communications, where maintaining stable service connection is more emphasized than other operation goals, and reveal that deploying Wi-Fi communication has a potential for improving the EE of the considered HetNet. Both simulation and experiment results validate the analytical derivations. These results together show that, compared with single-mode uplink transmission, the considered dual-mode transmission can achieve higher connection probability and EE simultaneously in LTE/Wi-Fi coexisting HetNets.

Index Terms—LTE/Wi-Fi, Carrier-Sense Multiple Access with Collision Avoidance (CSMA/CA), Energy Efficiency, Stochastic Geometry.

I. INTRODUCTION

Cellular communication employing dedicated licensed spectrum has been the norm of wireless industry for decades. To meet the ever-growing traffic demands arising from massive user equipments (UEs), however, the third-generation partnership project (3GPP) has recently developed long-term evolution (LTE) communication systems also in the unlicensed spectrum, which is to be shared with Wi-Fi networks [1]–[3]. As such, LTE networks with advanced physical layer techniques can benefit from the abundant unlicensed spectrum for providing high data rate. Moreover, dense heterogeneous networks (HetNets) formed by LTE and coexisting Wi-Fi networks can be exploited to achieve high area spectral efficiency with low communication latency, which has also been identified as a key technology candidate to enhance the fifth generation (5G) HetNets [4]–[6].

In order to utilize both licensed and unlicensed spectrum efficiently, various designs exploiting LTE/Wi-Fi coexisting HetNets have been proposed [7]–[13]. For example, the authors in [7] proposed a novel hyper-access point (HAP) framework, which enables UEs to access both LTE base stations (LBSs) and Wi-Fi access points (WAPs) for improved quality of service (QoS). The optimal contention window size for LTE/Wi-Fi coexisting HetNets was studied under non-saturated traffic condition in [8]. In [9] and [10], joint optimization of data packet splitting and radio access technology selection was investigated for maximizing the system throughput of LTE/Wi-Fi coexisting HetNets. Queue scheduling was investigated in [11] for improving the effective capacity of a three-tier network consisting of LTE, Wi-Fi, and device-to-device (D2D) communication links. Furthermore, a Nash bargaining game was proposed in [12] to maximize the achievable sum-rate for users accessing the unlicensed spectrum, and the interaction between network operators and users was analyzed in [13] using a multi-leader multi-follower Stackelberg game. Traffic offloading from LTE to D2D communication links using a cloudlet [14] and from LTE to Wi-Fi [15] were proposed for improving the performance of uplink transmissions.

Despite the fruitful development in the aforementioned works [7]–[15], two fundamental issues still remain unsolved for dual-mode uplink transmission in LTE/Wi-Fi coexisting HetNets. On the one hand, as specified in 3GPP Release 12 [1], the licensed and unlicensed spectrum are allocated...
for cellular and Wi-Fi communications in the uplink, respectively, only in an overlay manner. In this case, uplink Wi-Fi communications would not interfere with ongoing LTE communications such that protocol designs at dual-mode UEs can be simplified. However, the performance of dual-mode uplink communication in the LTE/Wi-Fi coexisting HetNets critically depends on the deployment of LBSs and WAPs as well as the adopted communication modes of UEs. For example, as the density of LBSs is usually lower than that of WAPs, it is more convenient for UEs located far away from the LBSs to access the WAPs whenever feasible. Moreover, different from LTE networks where the access of a given spectrum can be guaranteed by the system operator, Wi-Fi transmission is performed only opportunistically and hence, the likelihood of successfully connecting the WAPs may be limited. Therefore, in LTE/Wi-Fi coexisting HetNets, how to improve the successful connection probability for UEs employing dual LTE/Wi-Fi access is crucial but has not been reported in the literature yet.

On the other hand, energy efficiency (EE) of uplink transmission also a key performance metric for UEs of limited battery capacity [16]. Due to co-channel interference within the considered network, the EE of each user depends on the path loss and transmit power of all interfering users. To improve the EE of uplink transmission, an effective power control scheme is required to limit the maximum transmit power of all UEs. Recently, truncated channel inversion power control has been adopted in 3GPP LTE standard [1], which dynamically adjusts the transmit power of UEs such that the receiving power at LBSs meets the receiving threshold required for activating LBS receivers [17]–[24]. In [17], a dynamic pricing-based power control was proposed for maximizing the achievable sum-rate of UEs in an ultra-dense small-cell network. Moreover, power control of D2D links was investigated for interference mitigation in D2D communication underlying cellular systems in [18]. Distributed power control schemes were proposed in [19] and [20] for minimizing the total power consumption of uplink UEs and the average delay of packet delivery subject to several QoS constraints, respectively. To improve the system EE, joint transmit power control and beamforming was investigated for time division-duplexing (TDD) multiuser systems [21], [22]. The sum-rate, time stability, and spatial fairness of vehicular communication networks were studied assuming dynamic power control in [23], [24].

However, although power control for cellular communication has been well studied [17]–[24], power control for Wi-Fi communication and for LTE/Wi-Fi coexisting HetNets is rarely investigated. Different from cellular communication, Wi-Fi communication employs the carrier-sense multiple access with collision avoidance (CSMA/CA) scheme [25], [26], i.e., each WAP performs clear channel assessment (CCA) using carrier sensing and energy detection before information transmission. Thereby, power control for Wi-Fi communication is vital in not only CCA, but also information transmission. In particular, UEs employing a large transmit power during CCA may introduce exceedingly large interference to the WAPs, which saturates the energy detection circuits and even leads to the shut down of WAPs. Moreover, both the CSMA/CA scheme and power control can affect the EE and the successful connection probability of Wi-Fi communication simultaneously. For example, to compete for the Wi-Fi spectrum, a UE may employ a large transmit power to increase its communication range, but it may unfavorably increase the possibility of collision among contending UEs. Therefore, the transmit power of UEs and/or the receiving threshold at WAPs should be judiciously controlled for maximizing the performance of LTE/Wi-Fi coexisting HetNets.

To bridge the knowledge gap, in this paper, we propose a framework for modeling uplink transmissions in a power-controlled LTE/Wi-Fi coexisting HetNet, taking into account the maximal transmit power constraint of UEs. To comply with the 3GPP specifications [1], [27], we consider over- lay spectrum access for uplink transmission, where uplink cellular and Wi-Fi communications occupy the licensed and unlicensed spectrum, respectively. Different from the aforementioned literature on LTE/Wi-Fi coexisting HetNets [7]–[13], we consider UEs capable of jointly performing LTE/Wi-Fi dual access and dual power control. Thereby, the UEs first compete for connecting the WAPs as Wi-Fi UEs (WUEs) using CSMA/CA. If unsuccessful, they then attempt to establish cellular communication as cellular UEs (CUEs). In addition to the occupied communication modes, the UEs utilize truncated channel inversion power control1 with receiving signal thresholds chosen according to the associated LBSs/WAPs. We are interested in investigating how the considered dual-mode UEs are partitioned between unlicensed Wi-Fi and licensed cellular communications for different deployments of the HetNets as well as how would the formed user partition and the induced truncated inversion power control affect the connection probability of uplink spectrum access and the EE of uplink information transmission. To answer these questions, in this paper, we present a detailed analysis of the successful connection probability and the effective EE of LTE/Wi-Fi coexisting HetNets by utilizing the stochastic geometry tool. Moreover, we validate the derivations using both Monte Carlo simulations and experiment in a real testbed. Table I summarizes the detailed differences between the existing works and this paper in terms of performance metric, spectrum access and transmission schemes, and network deployment. We note that, while we focus on investigating LTE/Wi-Fi coexisting networks in this paper, the proposed dual-mode transmission scheme and the obtained analytical/simulation/experiment results are applicable for 5G networks and beyond as well [28]. In particular, by adopting a higher carrier frequency (e.g. 3.5 GHz) than 4G LTE (e.g. 2.1 GHz), 5G networks would suffer from more severe propagation losses during uplink data transmission. However, the maximum transmit power and the battery capacity of UEs remain almost unchanged, which significantly degrades the uplink connectivity and EE of 5G UEs. To tackle this challenge, the proposed cellular/Wi-Fi

1Advanced power control policies can also address the QoS requirements of specific applications but may not be afforded at users with limited battery and simplified hardware. In this case, power control of a simple form, e.g., the truncated power control policy, is preferred to balance between performance and complexity.
dual-mode transmission provides a promising approach. Our major contributions are as follows:

- We consider joint dual access and dual power control for UEs in the LTE/Wi-Fi coexisting HetNets and propose a stochastic geometry-based framework for analyzing its performance. By characterizing the cumulative distribution function (CDF) of the aggregated interference power, we derive the successful connection probability of Wi-Fi communication as well as tractable upper and lower bounds. Our results show that, to achieve the maximal successful connection probability, uplink power control of Wi-Fi communication should balance between mitigating truncation outage and resource competition in adaption to the density of UEs.

- The average EEs of uplink UEs having dual cellular and Wi-Fi access are evaluated and the derivations are validated using Monte Carlo simulations and real experiment. We reveal that, for UEs in Wi-Fi communication mode, their average EE saturates at large receiving signal thresholds, independent of the density of WAPs. In contrast, for UEs in cellular communication mode, there exists an optimal receiving signal threshold for maximizing their average EE.

- To analyze the impact of power control on both connection probability and EE in LTE/Wi-Fi coexisting HetNets, the effective EE of dual-mode UEs is further investigated. We reveal an interesting trade-off between the connection probability and the effective EE when LTE and Wi-Fi communications adopt monopolistic deployment. However, the considered dual LTE and Wi-Fi communication can achieve high EE and connection probability simultaneously.

The remainder of this paper is organized as follows. In Sec. II, the system model of power-controlled LTE/Wi-Fi coexisting HetNets in the uplink is presented in detail. The successful connection probability and EE of uplink transmission in Wi-Fi mode are analyzed in Sec. III. The numerical, simulation, and experiment results are presented in Sec. IV, where the impact of receiving thresholds, maximal transmit power, and deployment densities of UEs and LBS/WAPs on the system performance is investigated. Finally, Sec. V concludes the paper. For the sake of presentation, the key notations used in this paper are listed in Table II.

### II. System Model

#### A. Network Model

As shown in Fig. 1, we consider the uplink of a hybrid network consisting of WAPs deployed in coexistence with multi-tier LBSs. The LBSs of each tier and the WAPs are spatially distributed over the Euclidean plane. Let \( m_i^{(k)} \) be the location of LBS \( i \) in tier \( k \in \{1, \cdots, K\} \). For mathematical tractability, we assume that the locations of LBSs in tier \( k \), denoted by \( \Phi_k = \{ m_i^{(k)}; i = 1, 2, 3, \cdots \} \), follow a homogeneous Poisson Point Process (PPP) [29] with density \( \lambda_k \), independent of the LBSs in other tiers. Moreover, the locations of WAPs, denoted by \( \Phi_w = \{ w_i; i = 1, 2, 3, \cdots \} \), follow an independent homogeneous PPP with density \( \lambda_w \), where \( w_i \) is the location of WAP \( i \). The hybrid network serves UEs that are randomly distributed within the network. The spatial locations of the UEs, denoted as \( \Phi_{u} = \{ t_i \} \), follow a homogeneous PPP with density \( \lambda_u \), which is independent of \( \Phi_k \) and \( \Phi_w \). Due to the mobility of users, the occupied uplink channels and the load condition in the hybrid network may change from time to time. To simplify the performance analysis, wireless networks [30], [31]. Meanwhile, it has been shown in [32] that the homogeneous PPP provides an accurate approximation of the LBS deployment in real cellular networks.

#### TABLE I

<table>
<thead>
<tr>
<th>Reference</th>
<th>Contribution</th>
<th>Difference from our work</th>
</tr>
</thead>
<tbody>
<tr>
<td>[4]</td>
<td>Hyper-access point scheme to combine LBSs and WAPs for improving the QoS</td>
<td>Uplink power control is ignored in these works, and validating the results using real experiments is lacking.</td>
</tr>
<tr>
<td>[12]</td>
<td>Wireless energy transfer for maximizing the sum throughput in the network</td>
<td>These works considers downlink transmission rather than uplink transmission.</td>
</tr>
<tr>
<td>[5]</td>
<td>Optimal contention window size among LBSs and WAPs for traffic load balancing</td>
<td>The successful connection probability of UEs was ignored in these works.</td>
</tr>
<tr>
<td>[18], [19]</td>
<td>Joint transmit power control and beamforming for maximizing the system EE</td>
<td>These works considered power control for homogeneous networks rather than hybrid LTE/Wi-Fi networks.</td>
</tr>
<tr>
<td>[8]</td>
<td>Queue schedule design for maximizing the effective capacity of network</td>
<td></td>
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<tr>
<td>[9]</td>
<td>Nash bargaining game among different networks for maximizing the sum rate of users</td>
<td></td>
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<tr>
<td>[10]</td>
<td>Multi-ldr multi-follower Stackelberg game between network and users</td>
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<tr>
<td>[6], [7]</td>
<td>Optimal data packet splitting and radio access technology selection for maximizing the system throughput</td>
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<tr>
<td>[14]</td>
<td>Dynamic pricing-based power control for maximizing the achievable sum-rate of UEs</td>
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<tr>
<td>[16], [17]</td>
<td>Distributed power control for minimizing the total power consumption and the average delay</td>
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<tr>
<td>[18], [19]</td>
<td>Dynamic power control for balancing sum-rate and spatial fairness</td>
<td></td>
</tr>
<tr>
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</tbody>
</table>

2We note that the homogeneous PPP has been widely assumed for performance analysis of wireless networks [30], [31]. Meanwhile, it has been shown in [32] that the homogeneous PPP provides an accurate approximation of the LBS deployment in real cellular networks.
Fig. 1. Illustration of LTE/Wi-Fi coexisting HetNets, where active UEs can access a WAP or LBS depending on the dual-mode communication protocol. In contrast, UEs with large path loss are suspended for uplink transmission and hence, are inactive. Moreover, for successful energy detection, no active UE is allowed within a guard zone of each WAP.

analysis, we consider a worst-case load scenario3, where \( \lambda_u \) is large enough such that each LBS or WAP serves at least one associated UE per channel. That is, the uplink channels are fully occupied and the system is in a full-load situation [33], [34]. In general, derivation results under the full-load assumption define a lower bound of the system performance, whereas the performance gap vanishes when the number of UEs largely exceeds the number of WAPs and LBSs.

We consider both small- and large-scale propagation effects in the channel model. In particular, given a UE transmitting with power \( P_x \) at \( x \in \mathbb{R}^2 \), the receiving power at LBS/WAP \( y \in \mathbb{R}^2 \) is given by \( P_{x\rightarrow y} = P_x H_{x,y} L^{-1}(x,y) \), where

\[
H_{x,y} = S_{x,y} \|x - y\|^\alpha
\]

models the small-scale channel fading associated with multi-path propagation from \( x \) to \( y \). Besides, \( L(x,y) = S_{x,y} \|x - y\|^\alpha \) and shadowing \( S_{x,y} \), where \( \alpha \) is the path loss exponent and \( \|x - y\| \) denotes the Euclidean distance between \( x \) and \( y \). We consider Rayleigh fading and log-normal shadowing, i.e., \( H_{x,y} \sim \exp(1) \) is exponentially distributed with unit mean power and \( \log_{10}(S_{x,y}/10) \) is a zero-mean Gaussian random variable with variance \( N_0 \). For a tractable analysis, the path loss exponent \( \alpha \) is assumed to be identical within the hybrid network [35]–[38].

B. Dual-Mode Communication Protocol

The UEs employ dual LTE and Wi-Fi communications which occupy the licensed and the unlicensed spectrum, respectively, i.e., in an overlay mode [27]. For an individual UE, the preference of accessing WAPs is usually higher than the LBSs for reducing the potential charges associated with LTE services and lowering the energy consumption of uplink transmission. However, due to competitions and interference from other UEs, the UEs may fail to connect the Wi-Fi network. To minimize the service outage, we further assume that the failed UEs would reconnect the LTE network. This motivates the consideration of preference-based dual-mode access in this paper, whereby all UEs try to associate with WAPs first and, only if failed, then connect to LBSs.

In particular, before uplink information transmission starts, a potential uplink UE first sends a Request-To-Send (RTS) message to the WAPs. Each WAP performs carrier sensing and energy detection to determine whether a Wi-Fi connection is feasible [39]. The energy detection procedure measures the total receiving energy at the WAP, without knowing the 802.11 carrier signal sequences. If the total energy of all signals (including possibly interference and noise) received at the WAP exceeds a given threshold (for which it is coined “energy threshold” in the literature), the WAP will be shut down temporarily and suspends information reception. On the other hand, the carrier sensing procedure measures the signal strength of received 802.11 preamble symbols. If the signal power received at the WAP from an uplink user is below a given value i.e., the signal threshold, the WAP can neither sense the uplink user nor correctly decode its information. Otherwise, i.e., if the signal strength received at the WAP exceeds the receiving signal threshold during carrier sensing and, at the same time, the aggregated interference at the WAP is smaller than the receiving energy threshold during energy detection, the WAP replies a Clear-To-Send (CTS) message to the UE whereby the UE can start Wi-Fi communication. A collision occurs when multiple UEs attempt to communicate with the same WAP. To resolve the collision, each contending UE samples an independently and identically distributed (i.i.d.) random variable \( T_i \), i.e., the back-off timer, which is uniformly distributed in \([0, 1]\), and postpones its transmission by a time period equal to the realization of \( T_i \). In this case, the UE having the smallest back-off timer among its contenders is allowed to establish Wi-Fi communication [25], [39].

Let \( \Phi_{w_u} \) be the set of WUEs successfully establishing Wi-Fi communications. By employing the CSMA/CA scheme, different UEs would connect a WAP for Wi-Fi communication in a time-division, though random, manner. This implies that, under the full-load condition, each WAP is occupied by only one WUE in each time slot with high probability. As each WUE is randomly chosen from the UE set \( \Phi_u \) during CSMA/CA, the resulting WUE process can be modeled as a thinning of \( \Phi_u \) [29]. Therefore, \( \Phi_{w_u} \) is a PPP with density \( \lambda_w \).

On the other hand, UEs failing to connect the WAPs then attempt to associate with LBSs via cellular communication. We consider the biased-receiving-power (BRP) based association scheme4 [34]. Thereby, bias factor, \( B_k, k = 1, \ldots, K \), is assigned for the LBSs in tier \( k \). Each UE selects its association LBS in a given tier such that the biased receiving

4In the literature, users may also associate with either the nearest base station or the base station that maximizes the users’ receiving power/signal-to-interference ratio (SIR) [40]. The considered BRP based association scheme reduces to the receiving power based association when the same bias factor is employed across all tiers, i.e., \( B_1 = B_2 = \ldots = B_K \), and association with the nearest base station when the bias factors scale inversely with the BS transmit powers, i.e., \( B_k = 1/P_{B_k}, j = 1, \ldots, K \) [34]. On the other hand, due to its overwhelming computational complexity, the SIR based association is not considered in this paper.
signal power averaged over fading is maximized. Let \( P_{b,k} \) be the transmit power of LBSs in tier \( k \). The UE located at \( y \) is associated with the LBS at \( m(k) \) in tier \( k \) if and only if
\[
P_{b,k} B_k L_{\min,k}^{-1} (m(k), y) \geq P_{b,j} B_j L_{\min,j}^{-1} (m(j), y) \text{ for any } j \in \{1, \ldots, K\}.
\]
By adopting the BRP association rule, the probability that a UE is associated with LBSs in tier \( k \), denoted by \( A_k \), is given as [14]
\[
A_k = \frac{\lambda_k \mathbb{E} \left[ S_k^{2/\alpha} \right] B_k P_{b,k}^{2/\alpha}}{\sum_{j=1}^{K} \lambda_j \mathbb{E} \left[ S_j^{2/\alpha} \right] B_j P_{b,j}^{2/\alpha}},
\]
where \( \mathbb{E} [\cdot] \) is the expectation operator. We observe from (1) that the association probability jointly depends on the densities of BSs, the bias factors used for cell association, and the transmit powers in each tier. Moreover, the CUEs are assigned to different tiers of cells in proportion to factor \( \lambda_k \mathbb{E} \left[ S_k^{2/\alpha} \right] B_k P_{b,k}^{2/\alpha} \). While an WUE occupies the whole unlicensed spectrum during uplink information transmission, the CUEs access the licensed spectrum via orthogonal multiple access schemes such as OFDMA [21], where the licensed spectrum is divided into orthogonal subcarriers and the CUEs associated with the same LBS are assigned to different sub-carriers.

C. Uplink Power Control

Due to limited battery size and energy supply, the transmit power of UEs should be optimized to reduce the energy consumption for prolonging the battery lifetime [16]. For this purpose, the UEs employ the truncated channel inversion power control in adaption to the uplink transmission modes. Let \( P_{\max} \) be the maximum transmit power of the UEs. Each UE’s transmit power is adjusted to compensate the path loss such that the average signal power received at its associated LBS or WAP meets the required signal strength specified by the receiving signal threshold. Thereby, the UEs’ transmit power is given by \( P = \rho L(x, y) \), where \( \rho = \rho_c \) (or \( \rho = \rho_w \)) is the receiving signal threshold adopted for cellular (Wi-Fi) communication. We note that WAPs also employ \( \rho_w \) during carrier sensing. Moreover, by jointly considering uplink power control and energy detection, the path loss of the interfering UEs should exceed \( L_o = P_{\max}/\rho_c \) to prevent the WAP from being shut down [39]. As such, for the active WAPs, \( L_o \) defines a guard zone to protect them from excess network interference [41], since no WUE is allowed to exist within the protection zone, cf. Fig. 1 [41]–[43].

The receiving thresholds have different impacts on Wi-Fi and LTE communications. Since the WAPs perform physical-layer CCA before information transmission, the connection probability of Wi-Fi communication jointly depends on the receiving signal and energy thresholds. For instance, the access region for WAPs enlarges as \( \rho_w \) decreases, such that more UEs can compete for Wi-Fi spectrum access. However, employing a large receiving energy threshold, \( \rho_e \), at the WAPs may deteriorate the uplink UE due to large network interference. On the other hand, for LTE communications, the maximal output power \( P_{\max} \) and \( \rho_c \) limit the maximal tolerable path loss for each UE. When an UE cannot overcome the path loss to its associated LBS even if by transmitting at the maximal output power, its connection failure avoids and the UE is suspended from communication with the LBS. By adopting the truncated channel inversion power control, a potential UE is suspended from uplink transmission when it cannot connect to any BS. Therefore, the UEs are divided as active and inactive, depending on their channel conditions. Fig. 1 shows that, due to the limitation of maximal transmit power, a portion of the UEs at cell edge are inactive.

D. System Performance

By employing the preference-based dual-mode transmission, UEs accessing the hybrid network exhibit a unique partition between WUEs and CUEs, which is controlled by the receiving thresholds and the maximal transmit power during uplink power control. For a comprehensive performance evaluation, we consider both the successful connection probability of dual LTE and Wi-Fi access and the corresponding effective EE of the LTE/Wi-Fi coexisting HetNet.

1) Successful Connection Probability, \( P_{\text{succ}} \):
Define the successful connection probability of Wi-Fi communication as \( \xi = P[U = 1] \), where \( U \in \{0, 1\} \) is the random indicator of connection success for UEs implementing Wi-Fi communication. Moreover, let \( P_{\text{out}} \) be the truncation outage of cellular communication due to insufficient signal power received at LBSs. Based on Sec. II-B, the successful connection probability of UEs employing dual LTE and Wi-Fi access is given as
\[
P_{\text{succ}} = (1 - \xi)(1 - P_{\text{out}}) + \xi.
\]

2) Effective EE, \( \mathbb{E}[\eta] \): The effective EE of the LTE/Wi-Fi coexisting HetNet, is given as
\[
\mathbb{E}[\eta] = (1 - \xi)(1 - P_{\text{out}}) \mathbb{E}[\eta_w] + \xi \mathbb{E}[\eta_c],
\]
where \( \mathbb{E}[\eta_w] \) and \( \mathbb{E}[\eta_c] \) are the average EE of WUEs and CUEs, respectively. We note that, as Wi-Fi and cellular communications utilize orthogonal frequency spectrum in the overlay mode, the effective EE is a weighted sum of \( \mathbb{E}[\eta_w] \) and \( \mathbb{E}[\eta_c] \), where the weights are the connection probabilities of each communication mode. The average EE of WUEs is defined as
\[
\mathbb{E}[\eta_w] = \mathbb{E} \left[ \frac{W_w \log_2(1+\text{SINR}_w)}{P_d + P_r} \right],
\]
where \( W_w \) is the bandwidth of unlicensed spectrum allocated for uplink Wi-Fi communication, \( P = \rho_w L(o, t_0) \) is the transmit power of the typical WUE, \( P_s \) is the power consumption of RF circuits at the WUE, and \( \text{SINR}_w \) is the receiving signal-to-interference-plus-noise ratio (SINR) of the WAP serving the typical WUE. Likewise, the average EE of CUEs is defined as
\[
\mathbb{E}[\eta_c] = \mathbb{E} \left[ \frac{W_c \log_2(1+\text{SINR}_c)}{P_d + P_r} \right],
\]
where \( W_c \) is the total bandwidth of the licensed spectrum allocated for uplink cellular communication and \( N \) is the total number of uplink CUEs in a cell. \( P_{d,t} = \rho_e L(m; t_0) \) is the transmit power of the typical CUE, where \( m \) is the location of the LBS serving the typical CUE. Finally, \( \text{SINR}_c \) is the receiving SINR of the LBS.
TABLE II
LIST OF KEY NOTATIONS.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>Φ_k,λ_k</td>
<td>PPP of the kth tier LBS and the corresponding density</td>
</tr>
<tr>
<td>Φ_u,λ_u</td>
<td>PPP of the WAPs implementing Wi-Fi communications</td>
</tr>
<tr>
<td>Φ_w,u,w</td>
<td>PPP of WUEs and the corresponding density</td>
</tr>
<tr>
<td>α,P_max</td>
<td>Path loss exponent and maximal transmit power of UEs</td>
</tr>
<tr>
<td>P_u,P_v</td>
<td>Transmit power of UEs and receiving signal threshold of LBSs</td>
</tr>
<tr>
<td>α_w,P</td>
<td>Receiving signal and energy threshold of WAPs</td>
</tr>
<tr>
<td>H_u,w,S_x,y</td>
<td>Fading and shadowing between nodes located at x and y</td>
</tr>
<tr>
<td>W_u,W_v</td>
<td>Bandwidths for licensed and unlicensed spectrum</td>
</tr>
<tr>
<td>N</td>
<td>Number of uplink CUEs</td>
</tr>
<tr>
<td>B_k</td>
<td>Bias factor of LBSs in tier k</td>
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</table>

III. PERFORMANCE ANALYSIS OF LTE/Wi-Fi COEXISTING NETWORKS

In this section, we analyze the performance of the considered hybrid network in detail. We first derive the probability distribution functions (PDFs) of transmit power and aggregated interference for Wi-Fi communication in Sec. III-A. Based on the derivation results, the successful connection probability and the effective EE are further obtained in Sec. III-B and Sec. III-C, respectively.

A. PDFs of Transmit and Interference Powers

For a typical WAP located at the origin o, the propagation process from the typical WUE located at t_0 is defined as U_{w,u} ∼ Δ Φ_u,λ_u t_0 ∈ Φ_w,u,w. We note that U_{w,u} is obtained by mapping each point of the PPP Φ_u according to function L(α,t_0), and, hence, is also a homogeneous PPP with intensity measure A_{w,u}(x) = πλ_uE[S_u^δ] x^{2/α} [25, Theorem 2.34], where x = ∥t_0∥ is the Euclidean distance between the typical WUE and its serving WAP. Meanwhile, considering the maximum transmit power of UEs and the receiving signal threshold of WAPs, the maximum tolerable path loss for WUEs is given by L_{max} = L_{max} / ρ_u. Based on U_{w,u}, we characterize in Lemma 1 the PDF of the path loss between WUEs and WAPs.

Lemma 1: Under the maximum transmit power constraint, the PDF of the path loss between the typical WUE and its associated WAP is given by

\[ f_{L,w}(l) = \frac{\delta G_w l^{\delta-1} e^{-G_w l^\delta}}{1 - e^{-G_w L_{max}^\delta}}, \quad 0 \leq l \leq L_{max}, \]  

(2)

where \( \delta = 2/\alpha \) and \( G_w = \pi \lambda_u E[S_u^\delta] \).

Proof: Under the maximum output power constraint, the path loss distribution between the typical WUE and the corresponding WAP, conditioning on \( l \leq L_{max} \), is given by

\[ f_{L,w}(l) = \frac{f(l)}{P(l \leq L_{max})} = \frac{\delta G_w l^{\delta-1} e^{-G_w l^\delta}}{\int_{0}^{L_{max}} \delta G_w l^{\delta-1} e^{-G_w l^\delta} dl} = \frac{\delta G_w l^{\delta-1} e^{-G_w l^\delta}}{1 - e^{-G_w L_{max}^\delta}}, \]  

(3)

which completes the proof.

By employing the truncated channel inversion power control, the distribution of the transmit power at the typical WUEs is obtained in the following lemma.

Lemma 2: For the typical WUE employing the truncated channel inversion power control and receiving signal threshold \( \rho_u \), the PDF of the transmit power is given by

\[ f_P(x) = \frac{2G_w x^{\delta-1} e^{-G_w (\frac{x}{r})^\delta}}{\alpha \rho_u^\delta (1 - e^{-G_w L_{max}^\delta})}, \quad 0 \leq x \leq P_{max}. \]  

(4)

Moreover, the \( k \)th moment of the transmit power is given as

\[ \mathbb{E}[P^k] = \frac{\rho_u^k (k/\delta + 1, G_w L_{max}^\delta)}{(G_w L_{max}^\delta)^k (1 - e^{-G_w L_{max}^\delta})}, \]  

(5)

where \( k > 0 \) and \( \gamma(a,b) = \int_0^\infty x^{a-1}e^{-x}dx \) is the lower incomplete gamma function.

Proof: For Wi-Fi communication, the distance between the typical WUE and its corresponding WAP, denoted as \( r \), is Rayleigh distributed, i.e., its PDF is given by \( f(r) = 2G_w r e^{-G_w r^2} \). By employing the truncated channel inversion power control with receiving threshold \( \rho_u \), the PDF of transmit power \( P \) is obtained as

\[ f_P(x) = \frac{2G_w x^{\delta-1} e^{-\frac{G_w (x/r)^\delta}{\Pi (x \leq P_{max})}}}{\alpha \rho_u^\delta (1 - e^{-G_w L_{max}^\delta})}, \quad 0 \leq x \leq P_{max}. \]

The \( k \)th moment of \( P \) is given by \( \mathbb{E}[P^k] = \int_0^{P_{max}} x^k f_P(x) dx \), which completes the proof.

We note that, due to the distributed nature of Wi-Fi communication, the PDFs of path loss and transmit power in (2) and (4) are applicable for the interfering WUE-to-WAP communication links as well. In contrast, as cellular communication requires CUEs to associate with the LBS providing maximum BRP, the interfering CUEs are usually located further away from the typical LBS than the typical CUE. As such, the propagation process and the interference process of CUEs become correlated in the sense that, for a typical CUE, a large path loss would imply a small inter-cell interference on average [35]–[37], [44].

The aforementioned differences from cellular access motivate us to derive the distribution of the aggregated interference during Wi-Fi communication. We start with analyzing the aggregated interference for the typical WAP located at \( o \). According to Slivnyak’s theorem [36], the analytical results of the typical WAP can be extended to other WAPs in a straightforward manner. By employing the truncated channel inversion power control, the aggregate interference received at the typical WAP is given as

\[ I_w = \sum_{t_i \in \Phi_w,u \setminus \{t_0\}} P_{t_i} H_{t_i,o} L^{-1}(t_i), \]  

(6)

where \( P_{t_i} \) is the transmit power of the interfering WUE located at \( t_i \) and is a function of the distances between the interfering WUEs and their associated WAPs. We first derive the Laplace transform of the aggregated interference power for the Wi-Fi communication and the result is summarized in the following lemma.
Lemma 3: The Laplace transform of the aggregated interference received at the typical WAP is given as
\[ \mathcal{L}_{I_w}(s) = \exp \left(- \frac{\delta G_w s}{(1 - \delta)} \rho_w \left( \frac{P_{\max}}{P_e} \right)^{\delta-1} \cdot \mathcal{E}_P \left[ P_{2F_1} \left( 1, 1 - \delta; 2 - \delta; \frac{\rho_w s P}{P_{\max}} \right) \right] \right), \] (7)
where \( 2F_1 (; \cdot ; \cdot) \) is the Gauss hypergeometric function [44].

Proof: Please refer to Appendix A.

The CDF of the aggregated interference power received at the typical WAP, i.e., \( F_{I_w}(t) = \mathbb{P} \left( \sum_{t_i \in \Phi_{\mu_w} \setminus \{t\}} P_{t_i} H_{t_i,\alpha} |t_i|^{-\alpha} < t \right) \) can be calculated from (7) by performing the inverse Fourier transform of \( I_w \).

Lemma 4: The CDF of the aggregated interference power is given as
\[ F_{I_w}(t) = 1 - \frac{1}{2\pi i} \int_0^\infty \exp \left(-\delta \pi \lambda_w u^2 \left( \frac{P_{\max}}{P_e} \right)^{\delta-2} \mathcal{E}[P] \right) \cdot \exp (-ut) \left( e^{k_1(u)} - e^{k_2(u)} \right) \frac{du}{u}, \] (8)
where
\[ k_1(u) = G_{12}^{12} \left( \frac{\rho_w u e^{i\pi \mathcal{E}[P]}}{P_{\max}} \right) \left| \begin{array}{c} -1, \delta - 1 \\ -1, \delta - 2 \end{array} \right|, \]
\[ k_2(u) = G_{12}^{12} \left( \frac{\rho_w u e^{-i\pi \mathcal{E}[P]}}{P_{\max}} \right) \left| \begin{array}{c} -1, \delta - 1 \\ -1, \delta - 2 \end{array} \right|, \]
and \( G_{12}^{12}(\cdot | \cdot) \) is the Meijer’s G-Function.

Proof: Please refer to Appendix B.

B. Successful Connection Probability

Taking into account the CSMA/CA and the truncated channel inversion power control jointly, the random indicator of whether a UE can successfully perform uplink Wi-Fi communication is given by
\[ U_i = \mathbb{1} \left( P_{t_i} H_{t_i,\alpha} L^{-1} (t_i, w_i) \geq \rho_w \right) \cdot \mathbb{1} (I_w < \rho_e) \cdot \prod_{t_j \in \Phi_{\mu_w} \setminus \{t_i\}} \left( \mathbb{1} (T_i \leq T_j) + \mathbb{1} (T_i > T_j) \right) \] (9)
where \( T_j \) is the back-off time of contending device \( t_j \), \( P_{t_i} \) and \( P_{t_j} \) are the transmit power of UEs \( t_i \) and \( t_j \), respectively. \( L(t_i, w_i) \) is the large-scale attenuation from UE \( t_i \) to WAP \( w_i \). For successful Wi-Fi communication, i.e., \( U_i = 1 \), the first term in (9) ensures that, during carrier sensing, the receiving signal strength at WAP should exceed the receiving threshold \( \rho_e \). Moreover, the second term limits the aggregated interference power at the WAP from other UEs implementing Wi-Fi communications to be within the energy threshold \( \rho_e \) during energy detection. Furthermore, the third term ensures that the UE successfully competing for Wi-Fi communication should have the smallest back-off time among its contenders, unless the receiving signal strength from other contending UEs is smaller than the receiving threshold \( \rho_w \) [45].

Based on (9), the probability that an uplink UE can successfully establish Wi-Fi communications is derived in the following theorem.

Theorem 1: The success probability of performing Wi-Fi communication under the CSMA/CA and the truncated channel inversion power control is given by
\[ \xi = \left( 1 - e^{-G_w t_{\max}^2} \right) \Theta (\lambda_u, \rho_w) F_{I_w}(\rho_e), \] (10)
where
\[ \Theta (\lambda_u, \rho_w) = \frac{1 - \exp \left(-\delta \pi \lambda_u \mathcal{E}_P \left( P_{\max}^\delta \Gamma (\delta, \frac{P_{\max}}{P_e}) \right) \right)}{\delta \pi \lambda_u \mathcal{E}_P \left( P_{\max}^\delta \Gamma (\delta, \frac{P_{\max}}{P_e}) \right)}. \] (11)

Proof: Please refer to Appendix C.

Remark 1: Based on Theorem 1, the successful connection probability of Wi-Fi communication depend on \( \rho_w \) and \( \rho_e \) jointly. For given \( \rho_w \), when \( \rho_e \) is small, the term \( 1 - e^{-G_w t_{\max}^2} \) is close to 1 such that \( \xi \) increases at a rate of \( \Theta (\lambda_u, \rho_w) \). This result implies that, for small receiving signal threshold \( \rho_w \), the successful connection probability of Wi-Fi communication largely depends on the densities of UEs and WAPs. On the other hand, when \( \rho_w \) is close to \( P_{\max} \), \( \Theta (\lambda_u, \rho_w) \) is close to 1 and \( \xi \) decreases at a rate of \( 1 - e^{-G_w t_{\max}^2} \). Therefore, for large receiving signal threshold \( \rho_w \), the successful connection probability mainly depends on the density of WAPs. We observe that \( \xi < 1 \) always holds for any \( \rho_e \), implying that Wi-Fi networks cannot support all UEs to connect the WAPs. Meanwhile, for a given \( \rho_w \), \( F_{I_w}(\rho_e) \) increases with \( \rho_e \), i.e., a large \( \rho_e \) leads to a high successful connection probability of Wi-Fi communication, independent of \( \lambda_u \) and \( \lambda_w \).

However, \( F_{I_w}(\rho_e) \) in (8) lacks a closed-form expression, which hinders a convenient performance evaluation. To gain insight, we further derive a lower and an upper bound for \( F_{I_w}(\rho_e) \).

Lemma 5: For a given path loss exponent \( \alpha \), let \( \bar{F}_{I_w}(\rho_e) \) be an upper bound of \( F_{I_w}(\rho_e) \). We have
\[ F_{I_w}(\rho_e) \leq \bar{F}_{I_w}(\rho_e) = \exp \left(-G_w \rho_e^{-\delta} \Gamma (1 + \delta) \mathcal{E}[P^\delta] \right). \] (12)

Proof: Based on the definition of \( F_{I_w}(\rho_e) \) in (8), we have
\[ F_{I_w}(\rho_e) \leq \bar{F}_{I_w}(\rho_e) = \mathbb{P} \left( \max_{t_i \in \Phi_{\mu_w} \setminus \{t\}} P_{t_i} H_{t_i,\alpha} |t_i|^{-\alpha} < \rho_e \right), \]
where the last term gives the probability that the interference power from each interfering WUE is smaller than the energy detection threshold. Based on Lemma 1, we further have
\[ \mathbb{P} \left( \max_{t_i \in \Phi_{\mu_w} \setminus \{t\}} P_{t_i} H_{t_i,\alpha} |t_i|^{-\alpha} < \rho_e \right) = \exp \left(-G_w \rho_e^{-\delta} \mathcal{E}[H_{t_i,\alpha}] \mathcal{E}[P^\delta] \right) = \exp \left(-G_w \rho_e^{-\delta} \Gamma (1 + \delta) \mathcal{E}[P^\delta] \right), \]
which completes the proof.

Lemma 6: Let \( \underline{F}_{I_w}(\rho_e) \) be a lower bound of \( F_{I_w}(\rho_e) \). We have
\[ F_{I_w}(\rho_e) \geq \underline{F}_{I_w}(\rho_e) = 1 - \frac{\alpha}{2\pi}. \]
\[
\int_0^\infty \frac{\sin u}{u} \exp \left[-\frac{u^{\alpha/2} \rho_c}{[\kappa \sin \left(\frac{\pi}{2 \alpha}\right)]^{\alpha/2}} - u \cos \left(\frac{2\pi}{\alpha}\right) \right] du,
\]
(13)

where \( \kappa = G_w \mathbb{E} \left[ P^3 \right] \Gamma \left(1 + \delta\right) \Gamma \left(1 - \delta\right) \).

**Proof:** Please refer to Appendix D.

**Remark 2:** In the 3GPP specification, the receiving energy threshold \( \rho_c \) usually adopts a much larger value than the receiving signal threshold \( \rho_w \) to avoid frequent shut-down of the WAPs [2]. In this case, the resulting guard zone is small such that \( F_{L_w} (\rho_c) \) is usually close to its lower bound. We can observe from (13) that \( F_{L_w} (\rho_c) \) decreases with the density of WAPs, \( \lambda_w \), for a given \( \rho_c \). This result implies that dense deployment of WAPs would cause energy detection failure with high likelihood.

On the other hand, UEs failing in connecting the Wi-Fi networks would attempt to initiate cellular communications with the LBSs. The success probability of cellular communication is given by \( 1 - P_{\text{out}} \), where \( P_{\text{out}} \) is given as [21]

\[
P_{\text{out}} = \sum_{k=1}^{K} A_k \mathbb{P} \left\{ L \geq \frac{P_{\text{max}}}{\rho_c} \right\} = \sum_{k=1}^{K} A_k \exp \left[-G_{c,k} \left(\frac{P_{\text{max}}}{\rho_c}\right) \delta \right].
\]
(14)

Herein, \( G_{c,k} = \sum_{j=1}^{K} \pi \lambda_j \mathbb{E} \left[ S^j \right] \left( P_{b,j} / P_{b,k} \right) \delta \) is the superposition intensity of LBSs in all tiers normalized by the transmit power of LBSs in tier \( k \). \( A_k \) is the probability that a CUE is associated with a \( k \)-th tier LBS, c.f. (1). Based on (10) and (14), the success connection probability employing dual LTE/Wi-Fi access, c.f. Sec. II-D, is readily available.

**Remark 3:** From (14) we observe that the truncation outage probability increases with the receiving threshold of the LBSs. Hence, more CUEs will be suspended from service as \( \rho_c \) increases, similar to Wi-Fi communication. We note that the truncation outage event occurs more frequently for CUEs located at the cell edge. However, when \( \rho_c \) is small, \( P_{\text{out}} \) approaches zero, i.e., almost all CUEs can successfully associate and communicate with their serving LBSs. In contrast, only a portion of the devices can successfully communicate with WAPs even if \( \rho_w \) is small, since \( \xi < 1 \).

**C. Effective Energy Efficiency**

The effective EE captures the joint impact of receiving thresholds of WAPs and LBSs on the EE of dual-mode uplink information transmission, and is calculated based on the average EEs of cellular and Wi-Fi communications. Recall that the uplink average EE of Wi-Fi communication is defined as \( \mathbb{E} \left[ \eta_w \right] = \mathbb{E} \left[ \frac{W_{\text{ave}} \log_2 (1 + \text{SINR}_w)}{P_{\text{ave}}} \right] \), where the receiving SINR of the typical UWE is given as \( \text{SINR}_w = \frac{pH_{10, w}^{-\alpha}}{\kappa + \tau} \). Taking into account the PDFs of transmit and interference powers, the average EE can be obtained in the following theorem.

**Theorem 2:** The average EE of Wi-Fi communication, \( \mathbb{E} \left[ \eta_w \right] \), is given by (15) at the bottom of this page, where \( \tau = 2 \frac{P_{\text{ave}} (w + 1 + P_{\text{ave}})}{W_w} - 1 \) and \( \text{SINR}_w = \frac{P_{\text{ave}}}{\kappa + \tau} \).

**Proof:** Please refer to Appendix E.

When the path loss exponent \( \alpha = 4 \), i.e., \( \delta = \frac{1}{2} \), we have \( \delta_0 (\frac{1}{2})^2 F_1 (1, 1 - \delta; 2 - \delta; \alpha) = \sqrt{\alpha} \arctan \sqrt{\alpha} [36] \), whereby (15) further simplifies into (16). Based on (16), when \( \rho_w \) becomes large and \( \rho_c \gg \rho_w \), we have \( \arctan \left( \frac{\rho_w + P_{\text{ave}}}{P_{\text{ave}}} \right) \) approaching a constant and hence, the average EE of Wi-Fi communication saturates. For obtaining more insights, we further analyze the average EE of Wi-Fi communication in the large receiving threshold regime for general values of \( \beta \).

**Lemma 7:** When the receiving threshold of WAPs, \( \rho_w \), is close to \( P_{\text{max}} \), the average EE of Wi-Fi communication can be approximated by \( \mathbb{E} \left[ \eta_w \right] \) given in (18) at the top of next page.

**Proof:** When \( \rho_w \to P_{\text{max}} \), the path loss between a WUE and its associated WAP cannot exceed \( L_{\text{max}} = P_{\text{max}} / \rho_w \to 1 \) for successful Wi-Fi communication. According to (5), we have \( \mathbb{E} \left[ P^3 \right] \approx \frac{P^3}{\rho_w^3} \) as \( G_w L_{\text{max}}^3 \) is small. Meanwhile, since \( \rho_c \gg \rho_w \), the protection distance for Wi-Fi communication, i.e., \( L_p = P_{\text{max}} / \rho_c \), vanishes. Consequently, the Laplace transform of the aggregated interference power of interfering WUEs can be closely approximated by

\[
\mathbb{E} \left[ L_w \right] \approx \exp \left( -s^4 \rho_w^2 \Gamma (1 + \delta) \Gamma (1 - \delta) \right). \quad (17)
\]

Substituting (17) into (15), the approximate average EE of Wi-Fi communication can be obtained as given in (18), which completes the proof.

**Remark 4:** Lemma 7 and its proof reveal that, when the receiving threshold of WAPs becomes large, the access region allowed for the WUEs shrinks significantly. For example, we have \( L_{\text{max}} = P_{\text{max}} / \rho_w \to 1 \) and \( L_p \to 0 \) for \( \rho_w \to P_{\text{max}} \) and \( \rho_c \gg \rho_w \), whereby only WUEs located very close to their associated WAPs can successfully communicate. Interestingly, (17) and (18) show that both the aggregate interference power and the average EE of Wi-Fi communication in the large receiving threshold regime rarely change with the deployment density of WAPs. This is due to the truncated channel inversion power control, particularly to the effect of channel inversion for WUEs within such small access regions. Thereby, by increasing the density of WAPs, WUEs within the small access regions will drastically decrease their transmit power as their distances to the associated WAPs are reduced. In this case, the reduction in the transmit power of and the distance to the interfering WUEs counteract each other, leaving the aggregate interference power unchanged, c.f. (17). Moreover, given (17) and that the transmit power of WUEs is small, the density of WAPs can rarely impact the average EE of the Wi-Fi communication.

Moreover, recall that the average EE of cellular communication is defined as \( \mathbb{E} \left[ \eta_c \right] = \mathbb{E} \left[ \frac{W_c \log_2 (1 + \text{SINR}_c)}{(P_{\text{ave}} + P_{\text{ave}})} \right] \), where \( \text{SINR}_c \) is the receiving SINR at the CUE’s associated LBS. The average EE of cellular communication is given in the following lemma.

**Lemma 8 ([21]):** The uplink average EE of CUEs in cellular communication under the maximum transmit power constraint is given as

\[
\mathbb{E} \left[ \eta_c \right] = \sum_{k=1}^{K} A_k \sum_{n>0} P \left( N = n \right)
\]
path loss and the path loss of the interfering CUE for cellular communication, respectively.

Moreover, we assume

\[ L = \frac{c}{\tau_c} \]

where \( \tau_c \) is the tier of a CUE’s associated LBS.

\[
\mathbb{P}(N=n) = \frac{3.5^{3.5}}{(3.5)^n} \Gamma(n+3.5) \left( \frac{\lambda_c}{\lambda_p} \right)^{n+1} \left( \frac{3.5 + \lambda_c}{\lambda_p} \right)^{-n} \]  

\( \tau_c = \frac{c}{\min(p_c, \rho_c, \delta, \lambda_c, \lambda_p, w)} - 1, \)

and

\[
\mathbb{P}\{\text{SINR}_c > \tau_c | K=k, N=n\} = \frac{\delta G_k}{1 - \exp(-G_k L_{\text{max,c}}^\delta)} \cdot \int_0^{L_{\text{max,c}}} \int_0^{L_{\text{max,c}}} \exp(-G_k l_l^\delta - \frac{\tau_c \lambda_c}{\lambda_p} - \frac{\tau_c \lambda_p}{\lambda_c}) \sum_{j=1}^{K} \left( \frac{P_{k,j}}{\sum_{k=1}^{K} P_{k,j}} \right)^{1-\delta} A_j \cdot G_j \cdot \mathbb{E}[L_{\text{c}} | K=k] \left[ 1^{-\delta} F_1 \left( 1, 1-\delta, 2-\delta, -\frac{\tau_c \lambda_p}{\lambda_c} \right) \right] dl_l.

Moreover, \( L_{\text{max,c}} = P_{\text{max}} / \rho_c \) and \( l_l \) are the maximal tolerable path loss and the path loss of the interfering CUE for cellular communication, respectively.

Proof: Since the WUEs do not interfere with the CUEs in the considered system, the average EE of the successful CUEs can be derived in the same manner as [21] and the derivations are not repeated in this paper. Please refer to [21, Sec. IV] for details.

Fig. 2 evaluates the average EEs \( \mathbb{E}[\eta_w] \) and \( \mathbb{E}[\eta_c] \) in (15) and (19) as functions of the receiving thresholds \( \rho_w \) and \( \rho_c \) for different WAP and LBS densities, respectively, where we assume \( \rho_w = \rho_c = \rho \) and \( \alpha = 4 \). As the receiving signal threshold \( \rho \) increases, the WUEs need to transmit at a larger power such that the receiving signal power exceeds \( \rho \) after combating the path loss. However, under the maximum transmit power constraint, the WUEs with small path loss can communicate successfully with high likelihood and hence, the average uplink EE of WUEs increases. Meanwhile, we observe from Fig. 2 that, as \( \rho \) becomes large, the average EE of Wi-Fi communication saturates for all considered device densities. The reason for such behavior is two-fold. On the one hand, according to the simulation results in Fig. 5 of [21], the average transmit power under truncation power control saturates when the receiving threshold is large enough. On the other hand, the typical WUE and the interfering WUEs follow the same path loss distribution. Hence, the average transmission data rate and EE also are saturated when the receiving threshold of WAPs \( \rho \) is large. In Fig. 2, the average EE given in (18) is also illustrated, which closely approximates and provides an upper bound for the average EE in saturation.

Fig. 2 also shows that, the average EE of Wi-Fi and cellular communications exhibit drastically different behaviors rooted in their underlying communication paradigms. In particular, the cellular communication is controlled in a centralized manner such that information transmissions have to pass through the LBSs. Consequently, CUEs located at the cell edge are suspended for communication with high likelihood when they cannot overcome the path loss to their associated LBSs. In this case, by increasing the receiving threshold of LBSs, \( \rho_c \), more spectrum can be allocated to the active CUEs. However, as the distances between interfering CUEs and their associated LBSs also reduce with \( \rho_c \), the aggregated interference may increase as well. These two factors cause the average EE of cellular communication to increase slowly in the small \( \rho_c \) regime whereas it decreases in the large \( \rho_c \) regime. In contrast, all WUEs utilize the same unlicensed spectrum for Wi-Fi communication and the distances between the WUEs and their serving WAPs follow the same probability distribution. By increasing the receiving signal threshold of WAPs, the power consumption of WUEs required for overcoming the path loss between WUEs and their associated WAPs increases during information transmission. However, the density of WUEs keeps unchanged, which is still given by \( \lambda_w \), under the full-loading condition.

Remark 5: There is also an interesting trade-off between the successful connection probability and the EE. In particular, if Wi-Fi (cellular) communication is adopted for all devices in a monopolistic manner, a high EE (successful connection probability) would be achieved at a cost of compromised successful connection probability (EE). Hence, when both the successful connection probability and the EE are concerned, neither Wi-Fi nor cellular communication can dominate each other. Such connectivity versus EE perspective possibly explains why cellular communication cannot replace Wi-Fi communication, and vice versa, in the market. On the other hand, Wi-Fi and cellular communication in dual-mode transmission can simultaneously achieve high EE and successful connection probability, not to mention savings in charges, as they can
complement the individual technology for achieving efficient uplink transmission.

IV. PERFORMANCE EVALUATION

In this section, the derived analytical results are validated using Monte Carlo simulations, where the impact of the receiving signal and energy thresholds of LBSs and WAPs, maximal transmit power, and densities of nodes (UEs, LBSs, and WAPs) on the system performance is comprehensively evaluated. We consider a two-tier cellular network consisting of macro- and pico-cells in each tier, respectively. The cellular networks are further overlaid with an additional tier of WAPs, which provide Wi-Fi communication using the CSMA/CA scheme. The UEs with their associated LBSs and WAPs using the truncated channel inversion power control scheme while respecting the maximal transmit power constraint. Unless otherwise specified, the simulation parameters are set according to a typical LTE/Wi-Fi coexisting HetNet evaluated in [11], which are summarized in Table III. Note that the energy threshold, $\rho_c$, and the signal threshold, $\rho_w$, satisfy $\rho_c \gg \rho_w$ [2].

### A. Successful Connection Probability

We first evaluate the success probability of Wi-Fi and cellular communications in the considered HetNet. For convenience, we assume that the receiving signal thresholds of WAPs and LBSs satisfy $\rho_c = \rho_w = \rho$. Fig. 3 shows both the analytical (denoted as ‘Ana.’) and simulation results (denoted as ‘Sim.’) for the success probability of Wi-Fi and cellular communications, i.e., $\xi$ and $(1 - P_{\text{out}})$, as functions of the receiving signal threshold for different deployment densities of LBSs/WAPs, respectively. Moreover, the analytical and simulated evaluations, results from a small-scale experiment are provided to further validate the proposed scheme. The experiment measures the successful connection probability of cellular communications in a cellular/Wi-Fi coexisting HetNet, which comprises 5 LBSs (with density 0.15/km$^2$) and 20 WAPs (with density 0.15/km$^2$) deployed on the HUST campus. From Fig. 3 we observe that the analytical and the simulation/experiment results match well for different values of $\rho$, which validates the effectiveness of our derivations in Sec. III-B. Moreover, from Fig. 3(a) we observe that, for all considered WAP densities, the successful connection probabilities of UEs with small $\rho_w$ regime, but their values coincide with each other. Moreover, for a high density of WAPs $\lambda_w$, a large receiving signal threshold $\rho_w$ is required for achieving the maximal success probability $\xi^*$. In fact, the distances between devices and WAPs decrease with $\lambda_w$. Consequently, according to (10), by using a large receiving threshold, only UEs within short distances can connect with the WAPs and hence, the successful connection probability of UEs communicating with WAPs decreases. For cellular communication, we observe from Fig. 3(a) that, as the receiving signal threshold of the LBSs increases, the successful connection probability decreases from 1 to 0 quickly. This is because, by employing cellular communications, multiple CUEs within a cell are allowed to access the LBS simultaneously using orthogonal frequency spectrum allocation. Different from Wi-Fi communication, as the access region of LBSs decreases with $\rho_c$, the number of suspended CUEs increase with $\rho_c$ quickly. Therefore, to improve the successful connection probability, a small receiving signal threshold is required for cellular communication, whereas for Wi-Fi communication, the optimal receiving signal threshold has to be adapted to the density of WAPs. For performance comparison, we also evaluate the successful connection probability of a baseline scheme in Fig. 3(b), where each user randomly connects either the LTE or the Wi-Fi network with probability 1/2, i.e., each user randomly chooses a single-mode communication. We observe that the proposed dual-mode scheme can provide higher successful connection probability than the baseline scheme. This is because, if the uplink users cannot connect with the Wi-Fi network, they can reconnect with the cellular network, which ensures a large likelihood of successful connection by allocating orthogonal spectrum to each uplink user.

Fig. 4 evaluates the successful connection probability of Wi-Fi and cellular communications as functions of the densities of WAPs and LBSs for different densities of UEs and WAPs, i.e., $\lambda_w = \lambda_2$ and $\lambda_u = 2\lambda_2$, respectively. From Fig. 4 we observe that, as the density of LBSs increases, the success probability of cellular communication increases monotonically until reaching the upper limit 1, where UEs attempting to

\[
\xi = \frac{1}{\pi} \int_{\xi^*}^{1} \frac{\exp(-\frac{\rho \xi^*}{\rho_0})}{\pi \rho^2} d\rho.
\]

### TABLE III

PARAMETERS SETTING FOR SIMULATIONS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Default Value</th>
<th>Symbol</th>
<th>Default Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{\text{max}}$</td>
<td>30 dBm</td>
<td>$\lambda_1$</td>
<td>0.5 /km$^2$</td>
</tr>
<tr>
<td>$P_c$</td>
<td>6 dBm</td>
<td>$\lambda_2$</td>
<td>50 /km$^2$</td>
</tr>
<tr>
<td>$W_c, W_u$</td>
<td>8 MHz, 2 MHz</td>
<td>$\lambda_u$</td>
<td>1000 /km$^2$</td>
</tr>
<tr>
<td>$N_0$</td>
<td>$-90$ dBm</td>
<td>$P_{b,2}$</td>
<td>33 dBm</td>
</tr>
<tr>
<td>$P_{b,1}$</td>
<td>41 dBm</td>
<td>$s_1, s_2$</td>
<td>4 dBm</td>
</tr>
<tr>
<td>$\rho_c, \rho_u$</td>
<td>$-80$ dBm, $-80$ dBm</td>
<td>$\lambda_w$</td>
<td>5 /km$^2$</td>
</tr>
<tr>
<td>$P_{\text{out}}$</td>
<td>100</td>
<td>$\alpha$</td>
<td>4</td>
</tr>
<tr>
<td>$P_c$</td>
<td>1</td>
<td>$P_2$</td>
<td>1</td>
</tr>
</tbody>
</table>
Successful connection probability of Wi-Fi and cellular communication versus receiving thresholds $w$ and $c$ for different densities of WAPs and LBSs, respectively. (b) Successful connection probability of proposed dual-mode scheme and baseline scheme versus receiving thresholds $w$ and $c$ for different densities of WAPs and LBSs, respectively.

connect with the LTE network suffer from only negligible truncation outage. Hence, for LTE network employing orthogonal spectrum allocation, dense deployment of LBSs can enable almost all CUEs within the cell to successfully establish cellular communication. However, as the Wi-Fi transmission employs random spectrum access, the success probability of Wi-Fi communication is always smaller than that of cellular communication for the considered simulation setup. Moreover, different from LBSs, there exists an optimal deployment density of WAPs that maximizes the success probability of Wi-Fi communication. This is a result of employing energy detection at WAPs. In particular, to implement Wi-Fi communication, the aggregate interference at the WAPs should be smaller than the receiving energy threshold of the Wi-Fi network. As the aggregate interference increases with the density of WAPs, the success probability of Wi-Fi communication decreases when the density of WAPs is high enough. Therefore, unlike the LTE network whose successful connection probability monotonically increases with the density of LBSs, a dense deployment of WAPs may slightly penalize the success probability of Wi-Fi communication. This result imply that the deployment density of WAPs in the hybrid network should be cautiously determined to maximize the performance gains.

Fig. 5 shows the successful connection probability of Wi-Fi and LTE communications as functions of the maximal transmit power for different densities of UEs. As the maximal transmit power increases, more UEs can overcome the path loss and connect with LBSs and WAPs. Consequently, the success probability of LTE and Wi-Fi communications increase in
the low transmit power regime. However, when the transmit power becomes large, an excess aggregated interference power is received at both LBSs and WAPs. For Wi-Fi communication, however, the excess aggregated interference significantly decreases the successful connection probability, since more WAPs fail in energy detection. Therefore, there exists an optimal maximal transmit power for Wi-Fi communication to balance between uplink power control at UEs (i.e., for overcoming propagation path loss) and energy detection at WAPs. From Fig. 5 we also observe that the optimal maximal transmit power for Wi-Fi communication decreases with the deployment density of WAPs, i.e., a small transmit power is needed in dense Wi-Fi networks to achieve the maximal success probability of Wi-Fi communications.

B. Effective Energy Efficiency

Next, the effective uplink EE of the considered LTE/Wi-Fi coexisting HetNet is evaluated. Fig. 6 shows the effective EE as a function of the receiving signal threshold for different densities of UEs. From Fig. 6 we observe that the average EE of Wi-Fi transmission is higher than that of cellular communication for the considered simulation setup. This is because the density of WAPs is much higher than that of LBSs, whereby the transmission distances of Wi-Fi communications are much shorter than those of cellular communications. As a result, compared with only employing cellular communication, the LTE/Wi-Fi coexisting HetNet achieves a significantly higher effective EE without degrading the network coverage. From Fig. 6 we also observe that there exists an optimal receiving threshold for maximizing the average EE of cellular communication (or the LTE/Wi-Fi coexisting HetNet). This is because, for a small receiving threshold, the CUEs at the cell edge can access the network with ease, but reduce the average EE of cellular communication. On the other hand, for a large receiving threshold, the truncation outage events increase and deteriorate the average EE of cellular communication. Therefore, by adopting the optimal receiving threshold, the optimal spatial distribution of active users that balance between transmit power consumption and spatial information flows is needed for maximization of the effective EE.

Fig. 7 shows the effective EE as a function of the receiving threshold for different deployment densities of LBSs and WAPs. We observe from Fig. 7 that the effective EE of Wi-Fi communication increases with the density of WAPs for small receiving signal thresholds and saturates for large receiving signal thresholds. This is because employing a small signal threshold leads to a large access region of WAPs, which degrades the average EE of WUEs. Moreover, in the large receiving signal threshold regime, WUEs must be close to their corresponding WAPs to overcome the path loss and hence, the average EE increases. As the average EE of Wi-Fi
communication is higher than that of cellular communication, the optimal effective EE of the LTE/Wi-Fi coexisting HetNet increases with the density of WAPs. This result implies that dense deployment of WAPs can improve the maximal effective EE of hybrid Wi-Fi and cellular communication, though the success probability of Wi-Fi communication may be limited. Fig. 7 also compares the effective EE of the proposed scheme with that of the baseline scheme. We observe that the proposed dual-mode communication can achieve much higher effective EE than the baseline scheme, especially when the optimal receiving signal threshold is employed. This is because, as the receiving signal threshold increases, uplink UEs employing single-mode communications will be truncated with higher probability, which degrades the EE of UEs. However, the impact of truncation outage on the UEs with dual-mode communications is reduced as they can switch to another access network. Moreover, with the proposed scheme, each user tries to connect the Wi-Fi network with a high preference. Therefore, the proposed scheme can exploit the abundant Wi-Fi communication opportunities for much higher EE, compared to the baseline scheme.

Fig. 8 shows the effective EE as a function of the receiving threshold for different association schemes, which correspond to different settings of bias factors at the LBSs. In particular, we set i) $B_1 = B_2 = 1$, for the receiving power based association, ii) $B_1 = 1/P_1, B_2 = 2$, for the association with the nearest LBS, and iii) $B_1 = 0.1/P_1, B_2 = 10/P_2$ for the general BRP based association [34]. From Fig. 8 we observe that, as the receiving threshold increases, the average EE of cellular communications under all considered association schemes have almost the same value for the considered simulation setup. However, the effective EE of uplink users achieved by association with the nearest LBS is slightly higher than that by the receiving power based association. This is because, compared to the receiving power based association, the users associated with the nearest LBS have smaller communication distances and, hence, larger channel gains for uplink transmission, which can achieve the same data rate using lower transmit powers. Likewise, using a large bias for pico-cells, i.e., large $B_2$, can further improve the effective EE of uplink users, as the users failing to connect the WAPs can connect to the LBSs in the pico-cells for communication at small distances.

V. CONCLUSION

In this paper, the successful connection probability and the effective EE of an LTE/Wi-Fi coexisting multi-tier HetNet were analyzed for uplink dual-mode UEs performing truncated channel inversion power control. We showed that there exists an optimal receiving signal threshold for maximizing the success probability of Wi-Fi communication. Meanwhile, the average EE of WUEs saturates when the receiving signal threshold is significantly large. Dense deployment of WAPs was shown to enable a high EE uplink transmission but cannot provide reliable connection probability. However, Wi-Fi and cellular communications in the considered dual-mode transmission can enable highly energy-efficient and reliable uplink transmission simultaneously. Our analytical, simulation, and experiment results have shown that the introduction of Wi-Fi and cellular communications in the dual-mode transmission can significantly enhance the performance of the considered hybrid network, compared to single-mode Wi-Fi/cellular communications. These results pave the way to enhance the EE of cellular communication by exploiting coexisting Wi-Fi communication.

APPENDIX A
PROOF OF LEMMA 3

The Laplace transform of aggregated interference power at the typical WAP is

$$\mathcal{L}_I(w) = \mathbb{E}_{P(t), H_{t,\omega}} \left[ \exp \left( -s \sum_{i \in \Phi_{w, \omega} \backslash \{t_0\}} P_i |t_i|^\alpha H_{t,\omega} \right) \right]$$

$$= \mathbb{E}_{P(t)} \prod_{t_i \in \Phi_{w, \omega} \backslash \{t_0\}} \left( 1 + sp_i |t_i|^{-\alpha} \right)^{-1}$$

$$= \exp \left( -2G_w \mathbb{E}_P \left[ \frac{\int_0^{\delta} \frac{1}{r} \cdot \frac{(\frac{P_{\max}}{sP})^{\frac{1}{\alpha}} + r^{\alpha}/(sP)}{dr} \right] \right)$$

$$= \exp \left( -2G_w \mathbb{E}_P \left[ \int_0^{\delta} \frac{1}{t^{\alpha}} dt \right] \right)$$

$$= \exp \left( -\delta G_w \mathbb{E}_P \left[ \frac{P_{\max}}{\mathbb{E}_P} \right] \cdot \left( \frac{1}{(1 - \delta) \mathbb{E}_P} \right) \right)$$

where (a) is the probability generating functional (PGFL) of the PPP. We note that, due to the full-load assumption in Sec. II-A, the density of interfering WUEs is equal to the density of WAPs, $\lambda_w$. Moreover, (b) is obtained by substituting $t = r/(sP)^{1/\alpha}$ into (a). Finally, (c) is due to

$$\tau^\delta \int_{(x/r)^\delta} 1 \frac{1}{1 + x^{\frac{\alpha}{\delta}}} dx = \frac{1}{1 + \frac{\alpha}{\delta}} \left( \begin{array}{c} \frac{1}{\gamma} \left( 1 - 2 \right) \end{array} \right)$$

APPENDIX B
PROOF OF LEMMA 4

We calculate the inverse Fourier transform of $I_w$ in (B.1), where (d) reformulates the hypergeometric function using Meijer’s G-Function, i.e.,

$$\mathcal{F}^{-1} \left[ \mathcal{F} \left( \frac{\mathbb{E}_P}{\mathbb{E}_P} \right) \right] = \Gamma(c)_{\Gamma(a), \Gamma(b)} G^{12}_{22} \left[ x, \frac{-a-b}{1-c}, -1 \right]$$

Moreover, (e) is derived using a modified Bromwich contour as in [45] and [47], since the integration in (d) has a branch point at the origin. We note that, as the Meijer’s G-Function is an integral in the complex field, the directions of integrals are different between $G^{12}_{22} \left( \frac{\mathbb{E}_P e^{-ix} |P|}{\mathbb{E}_P} \right)$ and

$$G^{12}_{22} \left( \frac{\mathbb{E}_P e^{-ix} |P|}{\mathbb{E}_P} \right)$$

A branch point of a multi-valued function causes the function to be discontinuous when going around an arbitrarily small circle around this point.
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\[ F_{I_w}(t) = \frac{1}{2\pi t} \lim_{T \to \infty} \int_{-T}^{+iT} \frac{e^{st}}{s} \mathcal{L}_{I_w}(s) \, ds \]

\[ = \frac{1}{2\pi t} \lim_{T \to \infty} \int_{-T}^{+iT} \frac{1}{s} \exp \left[ -\delta G_w s \left( \frac{P_{\text{max}}}{\rho_c} \right) \delta^{-1} \right] \mathcal{E}_P \left[ P_2 F_l \left( 1, 1 - \delta; 2 - \delta; -\rho_c s P \frac{P_{\text{max}}}{P_{\text{max}}} \right) \right] \, ds \]

\[ \leq \frac{1}{2\pi t} \lim_{T \to \infty} \int_{-T}^{+iT} \frac{1}{s} \exp \left[ -\delta G_w s^2 \left( \frac{P_{\text{max}}}{\rho_c} \right) \delta \right] \mathcal{E}_P \left[ P_2 \Gamma(2 - \delta) \Gamma(1 - \delta - 1) \right] \mathcal{G}_{12}^{1/2} \left( \frac{\rho_c s^2 P \frac{P_{\text{max}}}{P_{\text{max}}}}{-1, \delta - 1} \right) \, ds \]

\[ = 1 - \frac{1}{2\pi t} \int_{0}^{\infty} \left[ \exp (-ut) \exp \left( -\delta G_w u^2 \left( \frac{P_{\text{max}}}{\rho_c} \right) \delta^{-2} \right) \mathcal{E}_P \left[ P_2 \right] \left( \exp \left( \mathcal{G}_{12}^{1/2} \left( \frac{\rho_c u^2 s P \frac{P_{\text{max}}}{P_{\text{max}}}}{-1, \delta - 1} \right) \right) \right] - \exp \left( \mathcal{G}_{12}^{1/2} \left( \frac{\rho_c u s P \frac{P_{\text{max}}}{P_{\text{max}}}}{-1, \delta - 2} \right) \right) \right] du \cdot \frac{1}{u}. \]

**APPENDIX C**

**PROOF OF THEOREM 1**

During energy detection, the aggregate interference power at the WAPs should be below the energy detection threshold \( \rho_c \), i.e., \( I_{I_w} < \rho_c \). According to the CDF of the aggregated interference power, we can obtain

\[ P \left( I_{I_w} < \rho_c \right) = F_{I_w}(\rho_c) = P \left( \sum_{t_i \in \Phi_{\text{WAP}}} P_{t_i} H_{t_i, o} |t_i|^{-\alpha} < \rho_c \right). \]

Based on (9), the success probability of Wi-Fi communication can be rewritten as

\[ \xi = P \left[ U_1 = 1 \right] = P \left[ U_1 \right] = \xi_1 \cdot \xi_2 \cdot F_{I_w}(\rho_c). \]

Herein, \( \xi_1 \) gives the probability that the device located at \( t_i \) is allocated a sufficient transmit power to overcome the path loss without violating the maximum transmit power,

\[ \xi_1 = P \left( \left\{ t_i \left( H_{t_i, o} |t_i|^{-\alpha} > \rho_c \right) \right\} \right) = \int_{0}^{L_{1/\alpha} \max} 2G_w x e^{-G_w x^2} \, dx = 1 - e^{-G_w L_{\max}^{1/\alpha}}, \]

where the second holds because, for a generic Wi-Fi transmit, the distance \( |t_i - w_i| \) has the Rayleigh distribution \( f(x) = 2G_w x e^{-G_w x^2}, \quad 0 \leq x < +\infty \). Consequently, the transmitter power for the WUE is given by \( P = \rho_w x^\alpha \), \( 0 \leq P \leq P_{\text{max}} \).

Moreover, \( \xi_2 \) gives the probability that device \( t_i \) is allocated the spectrum required for Wi-Fi communication under the CSMA/CA scheme,

\[ \xi_2 = E_{T, L, P_{t_i}} \left( \prod_{t_j \in \Phi_{t_i}} \left( 1 \left( T_i \leq T_j \right) + 1 \left( T_i > T_j \right) \right) \cdot I \left( P_{t_j} H_{t_j, o} |t_j|^{-\alpha} < \rho_w \right) \right) \]

\[ = \int_{0}^{1} E_{L, P_{t_i}} \left( \prod_{t_j \in \Phi_{t_i}} \left( 1 - x \exp \left( -\rho_w L \left( t_j, w_i \right) \right) \right) \right) \, dx \]

\[ = \int_{0}^{1} \exp \left( -2\pi \lambda \int_{0}^{L_{1/\alpha} \max} \exp \left( -\frac{\rho_w y^\alpha}{P_{t_i}} \right) \, x \, dy \right) \, dx \]

\[ = \int_{0}^{1} \exp \left( -\frac{\delta \pi \lambda x}{\rho_w} \mathcal{E}_P \left( P \int_{0}^{P_{\text{max}}} \exp \left( -z \right) z^{\delta - 1} \, dz \right) \right) \, dx \]

**APPENDIX D**

**PROOF OF LEMMA 6**

To obtain the lower bound, we note that the aggregated interference power is larger than the energy detection threshold when an interfering device is close to the typical WAP. Therefore, we have

\[ F_{I_w}(\rho_c) = P \left( \sum_{t_i \in \Phi_{t_i}} P_{t_i} H_{t_i, o} |t_i|^{-\alpha} < \rho_c \right) > \|

\[ P \left( \sum_{t_i \in \Phi_{t_i}} P_{t_i} H_{t_i, o} |t_i|^{-\alpha} < \rho_c \right), \]

where \( \Phi_{t_i} \) is the set of interfering devices in the Euclidean plane. Following the same approach as Lemma 3, the Laplace transform of the aggregated interference power of WUEs in the Euclidean plane can be obtain as

\[ \mathcal{L}_{I_w}(s) = E \left[ \exp \left( -s \sum_{t_i \in \Phi_{t_i}} P_{t_i} H_{t_i, o} |t_i|^{-\alpha} \right) \right] \]

\[ = E \left[ \prod_{t_i \in \Phi_{t_i}} \frac{1}{1 + s P_{t_i} |t_i|^{-\alpha}} \right] \]

\[ = \exp \left( -2G_w \mathcal{E}_P \left( \int_{0}^{1 + r^{\alpha} / (s P_{t_i})} r \, dr \right) \right) \]

\[ = \exp \left( -G_w \mathcal{E}_P \left( \int_{0}^{(s P_{t_i})^{1/\alpha}} 1 + y^{\alpha/2} \, dy \right) \right) \]

\[ = \exp \left( -G_w s^{1/\alpha} \mathcal{E}_P \left( -G_w s^{1/\alpha} \mathcal{E}_P \left( \frac{P_{\delta}}{(1 + y^{\alpha/2})} \right) \right) \right), \]

where \( (h) \) follows by substituting \( t = r^{\alpha} / (s P_{t_i}) \) and \( \int_{0}^{(s P_{t_i})^{1/\alpha}} 1 + y^{\alpha/2} \, dy = \Gamma(1 + \delta) \Gamma(1 - \delta) \). Moreover, by inverse
Fourier transform, we obtain

\[
F_{xw}(t) = \frac{1}{2\pi t} \lim_{T \to \infty} \int_{-iT}^{iT} e^{st} e^{i\omega t} L_{xw}(s) \, ds
\]

\[
= \frac{1}{2\pi t} \lim_{T \to \infty} \int_{-iT}^{iT} \exp(st - \kappa s^2) \, ds
\]

\[
= -\frac{1}{2\pi t} \alpha \exp \left[ \frac{-u^2}{2t} \left( \frac{2\pi}{\alpha} \right)^{3/2} - u \cot \left( \frac{2\pi}{\alpha} \right) \right] \sin u \, du.
\]

(D.2)

whereby the lower bound of \( F_{xw}(\rho_c) \) is proved.

**APPENDIX E**

**PROOF OF THEOREM 2**

From (15) we have

\[
\mathbb{E}[\eta_w] = \int_0^{\infty} P\{\text{SNR}_{w} > \tau\} \, d\eta_w
\]

\[
= \int_0^{\infty} P\{\text{SNR}_{w} > \tau\} \, d\eta_w
\]

\[
= \left\{ \frac{H_{1,0}}{\text{SNR}^2} + \sum_{t_i \in \Phi_{w \setminus \{t\}}} H_{1,0} L_1 (t_i, w_1) L_{t_i}^{-1} \right\} > \tau
\]

\[
= \left\{ \frac{H_{1,0}}{\text{SNR}^2} + \sum_{t_i \in \Phi_{w \setminus \{t\}}} H_{1,0} L_1 (t_i, w_1) L_{t_i}^{-1} \right\}
\]

\[
= \left\{ \exp(-\tau \text{SNR}^2) + \sum_{t_i \in \Phi_{w \setminus \{t\}}} H_{1,0} L_1 (t_i, w_1) L_{t_i}^{-1} \right\}
\]

\[
= \left\{ \exp(-\tau \text{SNR}^2) \right\} \cdot L_{xw}(\tau),
\]

(E.1)

where \( N_o \) is the received noise power and \( L_{xw}(\tau) \) is the Laplace transform of the total interfering power. Considering the maximal transmit power of WUEs, we have

\[
\mathbb{E}[\exp(-\tau \text{SNR}^2)] = \int_0^{\infty} \exp\left( -\left( \frac{2}{\text{SNR}^2} \right) - 1 \right) \text{SNR}^{-1} f_{Lw}(l) \, dl,
\]

(E.2)

where \( f_{Lw}(l) \) is given in Lemma 1. Moreover, \( L_{xw}(\tau) \) can be calculated from Lemma 3 as

\[
L_{xw}(\tau) = \int_0^{L_{\text{max}}} f_{Lw}(l) \exp\left( \frac{\delta G_w \tau}{1 - \delta} \frac{P_{\text{max}}}{\rho_c} \right)^{-1} \cdot P_{2F_1} \left( l, 1 - \delta; 2 - \delta; -\frac{\rho_c}{P_{\text{max}}} \right) \, dl.
\]

(E.3)

Finally, (15) can be obtained by substituting (E.2) and (E.3) into (E.1).

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